Expectation of $N_x(0,1)$

$$E[X] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} \exp\left(\frac{1}{2}x^2\right) dx =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x \exp\left(\frac{1}{2}x^2\right) dx =$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} x \exp\left(\frac{1}{2}x^2\right) dx + \int_{0}^{\infty} x \exp\left(\frac{1}{2}x^2\right) dx\right] =$$

$$= \frac{1}{\sqrt{2\pi}} \left[-\int_{0}^{\infty} x \exp\left(\frac{1}{2}x^2\right) dx + \int_{0}^{\infty} x \exp\left(\frac{1}{2}x^2\right) dx\right] = 0$$

Constant

$$I = \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx$$

$$I^2 = \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx \int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{2}\right) dy =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2+y^2}{2}\right) dxdy = (*)$$

Polar coordinates

$$x = r\cos(\varphi)$$

$$y = r\sin(\varphi)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \to \int_{0}^{\infty} \int_{0}^{2\pi}$$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = r\cos^2(\varphi) + r\sin^2(\varphi) = r$$

$$(*) = \int_{0}^{\infty} \int_{0}^{2\pi} \exp\left(-r^2/2\right) r d\varphi dr = 2\pi \int_{0}^{\infty} \exp\left(-r^2/2\right) = (**)$$

substitution

$$r^2 = z; \;\; 2rdr = dz; \;\; \int_{0}^{\infty}$$

$$(**) = 2\pi \int_{0}^{\infty} z \exp(-z)\, dz = 2\pi$$

$\to$

$$I = \sqrt{2\pi}$$