How intervals and tests differ

As mentioned, confidence interval for μ with known σ^2 has statistics $\bar{x}$ and it is

$$I_{\bar{x}} = \left(\bar{x} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}},\, \bar{x} + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right)$$

where all is in real scale.

The test is done in a standardize scale. Here:

Confidence interval (region of acceptance) is

$$\left(-z_{\frac{\alpha}{2}},\, z_{\frac{\alpha}{2}}\right)$$

Critical region (supplement to confidence interval) is

$$W = \left(-\infty,\, \bar{x} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right) \cup \left(\bar{x} + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}},\, \infty\right)$$

Statistics (normalized) is

$$T_t = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - \mu_0}{\sigma} \sqrt{n}$$

Remark

Here the real scale plays no role and the interval is simple.