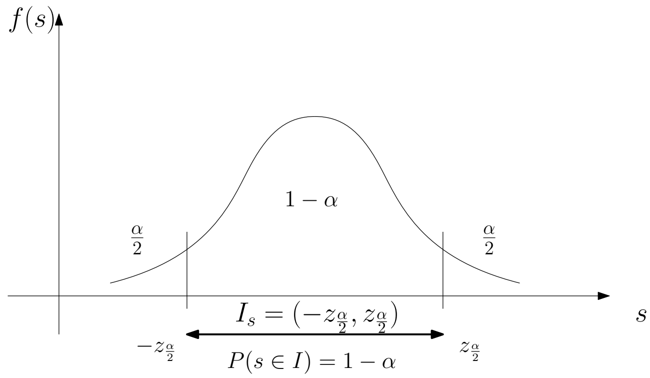
How to come from data to CI

Let us have data from $N_x\left(\mu, \sigma^2\right)$ where σ^2 is known, we want CI for μ with the sample of the length n.

We start with **standard normal distribution** of s - standard normal variable



To obtain our data with **expectation** μ **and variance** σ^2 we need to use transformation

$$x = \mu + \sigma s \;\rightarrow\; \mu = x - \sigma s$$

Then

$$-z_{\frac{\alpha}{2}} \rightarrow x + \sigma z_{\frac{\alpha}{2}} \;\; \text{and} \;\; z_{\frac{\alpha}{2}} \rightarrow x - \sigma z_{\frac{\alpha}{2}}$$

and the interval $I_s \rightarrow I_x$

$$I_x = \left(x - \sigma z_{\frac{\alpha}{2}},\; x + \sigma z_{\frac{\alpha}{2}}\right).$$

However, we are not interested in interval for x but in interval for the **parameter** μ with the statistics $\bar{x}$. That is, we must work with the distribution of the statistics $\bar{x}$.

We know

$$E\left[\bar{x}\right] = \mu \;\; \text{and} \;\; D\left[\bar{x}\right] = \frac{\sigma^2}{n},$$

i.e. we must replace $\sigma \rightarrow \frac{\sigma}{\sqrt{n}}$ ($n$is length of the sample). Then the interval I_x becomes the confidence interval $I_{\bar{x}}$

$$I_{\bar{x}} = \left(\bar{x} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}},\; \bar{x} + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right).$$