Important tests

1 z-test

normal statistics $z \sim N(0,1)$

Use:

– test of expectation with known variance

2 t-test

Student statistics $t \sim St(\nu)$

Use:

– test of expectation with unknown variance
– test of regression coefficient

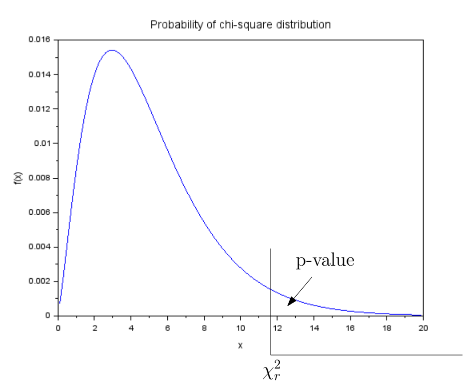
3 chi-square test

For discrete or discretized variables.

Statistics

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(\nu)$$

where O_i are observed and E_i are expected absolute frequencies.



If $\chi^2 = 0$, p-value= 1 - $V_E = 0$ nothing is explained.
If $\chi^2 \to \infty$, p-value$\to$ 0 - $V_U \to 0$ all is explained.

Use:

– test of variance
– test of independence

4 F-test

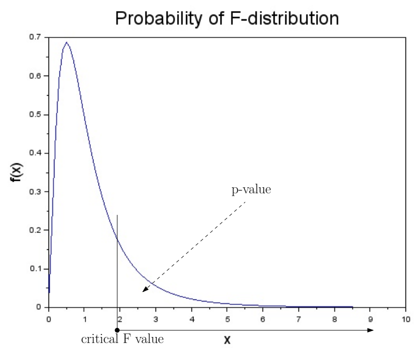
Test of explained and unexplained variance.

Statistics

$$F = \frac{D_{\mathrm{E}}}{D_{\mathrm{U}}} \sim F(\nu_1, \nu_2)$$

Use:

– test of two variances
– ANOVA
– test of prediction in regression analysis



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ANOVA I

We have data from several sources (populations). We test, if the expectations of the populations are equal.

The data are x in the following table as absolute frequencies

X_1	X_2	X_3
x	x	x
x	x	x
x	x	x
$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$
s_1^2	s_2^2	s_3^2

We compute averages $\bar{x}$ and variances D. Then:

- Average of variances s_i^2 correspond to **unexplained variance** D_U - it describes the overall variance in the data.
- Variance of the averages $\bar{x}_i$ corresponds to **explained variance** D_E - it expresses the variance between classes.

If the explained variance D_E is sufficiently larger with respect to the unexplained one D_U then we conclude that the classes are not equal.

Statistics: $F = \frac{D_E}{D_U} \sim F$ distribution (right-sided test)

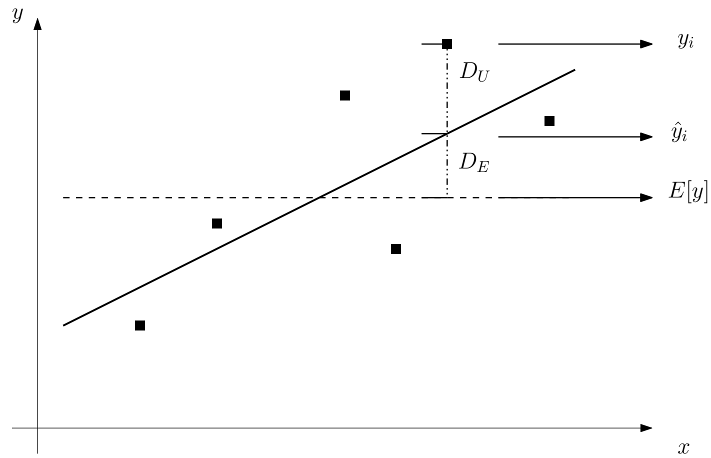
H0: are equal.
HA: are not equal.

ANOVA II

	X_1	X_2	X_3		
Y_1	x	x	x	$\bar{y}_1$	s_{y1}^2
Y_2	x	x	x	$\bar{y}_2$	s_{y2}^2
Y_3	x	x	x	$\bar{y}_3$	s_{y3}^2
Y_4	x	x	x	$\bar{y}_4$	s_{y4}^2
	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$		
	s_{x1}^2	s_{x2}^2	s_{x3}^2		

Two tests - first for columns and second for rows.

Regression



Friedman test

Tests if the quality of several objects is the same. It does not take into account the differences of those who rate.

H0: is equal.
HA: Is not equal.

Example

A quality of five shops is checked by three evaluators. Each evaluator rates each shop. The results are in the table

Evaluator/shop	1	2	3	4	5
1	x	x	x	x	x
2	x	x	x	x	x
3	x	x	x	x	x

We are interested in evaluation of the shops (not evaluators).

Remark: The shops are called treatment, the evaluators are subjects. In the Statext, we choose "Each data set is for subject" which means, the data in rows come from individual subjects.

χ^2-test of distribution

Tests, if the distribution is as expected. Works for discrete data or continuous ones discretized on intervals.

It is based on

- O observed absolute frequencies - from measured data,
- E expected absolute frequencies - constructed so that:
  - H_0 is precisely fulfilled,
  - number of data (sum of frequencies) is the same as for measured ones.

Statistics with χ^2 distribution is

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

H0: yes, the distribution is as expected.
HA: no, it is not.

Example

Test if the number of traffic accidents is uniformly distributed during the week, if we measured the following number of accidents

Weekdays	Saturday	Sunday
587	98	103
O_1	O_2	O_3

Sum $\sum O_i = 788$. Time axis has 7 intervals (days). Weekdays has 5, Sat and Sun have 1. So we need to divide 788 into groups with proportion 5/1/1.

$$E_1 = \tfrac{788}{7}5 = 562.86, \quad E_2 = \tfrac{788}{7}1 = 112.57, \quad E_3 = \tfrac{788}{7}1 = 112.57$$

$$\chi^2 = 3.73; \quad pv = 0.155$$

As $pv > 0.05$ the H_0 is not rejected at the confidence level $\alpha = 0.05$.

χ^2-test of independence

Tests independence of two variables from normal distribution. It is similar to the previous one. Differs in computation the frequencies.

H0: are independent.
HA: are not independent.

Example

For random variables x and y we have the frequency table

$$T = \begin{array}{c|ccc} x\backslash y & 1 & 2 & 3 \\ \hline 1 & 15 & 11 & 8 \\ 2 & 27 & 18 & 21 \end{array}$$

Test their independence.

1. Normalize to probability

$$f(x, y) = T / \sum(T)$$

   sum = 100;

$$f(x, y) = \begin{bmatrix} 0.15, & 0.11, & 0.08 \\ 0.27, & 0.18, & 0.21 \end{bmatrix}$$

2. Compute marginals

$$f(x) = \sum_y f(x,y), \quad f(y) = \sum_x f(x,y)$$

$$f(x) = [0.42,\ 0.29,\ 0.29], \quad f(y) = \begin{bmatrix} 0.34 \\ 0.66 \end{bmatrix}$$

3. Do the product of marginals

$$g = f(y) \times f(x)$$

$$\begin{bmatrix} 0.34 \\ 0.66 \end{bmatrix} [0.42,\ 0.29,\ 0.29]$$

4. Return to absolute frequencies - multiply by the sum

$$E = g \sum T$$

Pearson and Spearman test of independence

Pearson test tests the correlation coefficient

$$R = \frac{C[X,Y]}{\sqrt{D[X]\,D[Y]}}.$$

If $R = 0$ the random variables X and Y are uncorrelated. If $R \to -1$ or 1, the variables are strongly correlated. The test is both sided.

Its statistics is the sample correlation coefficient

$$r = \frac{\operatorname{cov}(x,y)}{\sqrt{\operatorname{var}(x)\operatorname{var}(y)}} \sim t \text{ Student distribution}$$

H0: The variables are independent, i.e. $r = 0$.
HA: $r \neq 0$

For p-value small, the independence is rejected - variables are dependent.

Spearman test is a nonparametric variant of Pearson. Instead of data it uses their ranks. Can be used for discrete variables.

McNemar test

We measure binary data (yes/not) before and after some action. We test if the action caused any change.

Example

We ask 20 people if they have a cold. The answers are yes/not. Then we give them some medicine and ask again (in the same order - paired test). The question is whether there was any change after the application of the drug (either positive or negative).

Data are set in the table

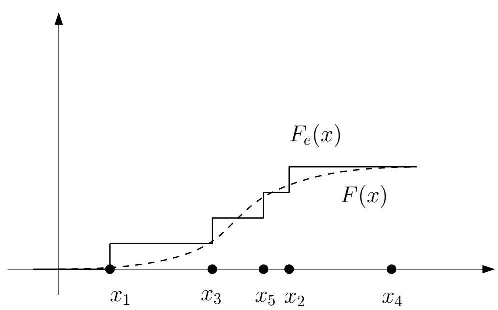
before\after	no	yes
no	31	15
yes	27	27

H0: no change.
HA: there is some change.

→ if p-value is small, a change has been detected.

Kolmogorov-Smirnov test

Tests if the distribution is as expected. Based on
– tested distribution
– empirical distribution from sample



Statistics is absolute value of the maximal difference between F and F_e.