Multivariate regression model

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u_t + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_t + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_t$$

– regression vector $\psi_t = [y_{t-1}, u_t, v_t, 1]$

– parameters $\theta = \{a, b, c, k\}$,

– r covariance matrix e_t.

It can be written as

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t - \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u_t - \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_t + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_t =$$

$$= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, - \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, - \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}, - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \right\} \begin{bmatrix} y_{1;t} \\ y_{2;t} \\ y_{1;t-1} \\ y_{2+t-1} \\ u_t \\ v_{1;t} \\ v_{2;t} \\ v_{3;t} \\ 1 \end{bmatrix}$$

From it, we can derive the extended regression vector as

$$\Psi = [y_{1;t}, y_{2;t}, y_{1;t-1}, y_{2;t-1}, u_t, v_{1;t}, v_{2;t}, v_{3;t}, 1]'$$

and the corresponding matrix of regression coefficients is

$$\theta = \begin{bmatrix} 1 & 0 & a_{11} & a_{12} & b_1 & c_{11} & c_{12} & c_{13} & k_1 \\ 0 & 1 & a_{21} & a_{22} & b_2 & c_{21} & c_{22} & c_{23} & k_2 \end{bmatrix}$$

The information matrix and its update now will be

$$V_t = V_{t-1} + \begin{bmatrix} y_{1;t} \\ y_{2;t} \\ y_{1;t-1} \\ y_{2;t-1} \\ u_t \\ v_{1;t} \\ v_{2;t} \\ v_{3;t} \\ 1 \end{bmatrix} [y_{1;t}, y_{2;t}, y_{1;t-1}, y_{2;t-1}, u_t, v_{1;t}, v_{2;t}, v_{3;t}, 1] = \begin{bmatrix} V_y & V_{yp} \\ V_{yp}' & V_p \end{bmatrix}$$

From this it can be seen that

$$V_y = [\cdot]_{2\times 2} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}, \; V_{yp} = [\cdot]_{2\times 7}, \; V_p = [\cdot]_{7\times 7}$$

The estimate of parameters is standard

$$\hat{\theta} = V_p^{-1} V_{yp}$$

and the estimate will be a row vector.

Especially for **constant model** we have

– extended regression vector

$$\Psi = [y_{1;t}, y_{2;t}, 1]'$$

– matrix of parameters

$$\theta = \begin{bmatrix} 1 & 0 & k_1 \\ 0 & 1 & k_2 \end{bmatrix}$$

– partitioned information matrix

$$V_y = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}, \; V_{yp} = \begin{bmatrix} \cdot & \cdot \end{bmatrix}, \; V_p = \cdot$$

– estimate

$$\hat{\theta} = V_{yp}/V_p$$