Quadratic form

$$[x_1, x_2, x_3]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$= \left[a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3\right]$$

$$\ldots$$

	x_1	x_2	x_3
x_1	a_{11}	a_{12}	a_{13}
x_2	a_{21}	a_{22}	a_{23}
x_3	a_{31}	a_{32}	a_{33}

Criterion for $2nd$ order regression model in the state-space form

$$J = (y_t - s_t)^2 + \omega u_t^2 + \lambda\left(u_t - u_{t-1}\right)^2 =$$

$$= y_t^2 - 2y_ts_t + s_t^2 + \omega u_t^2 + \lambda u_t^2 + 2\lambda u_tu_{t-1} + \lambda u_{t-1}^2$$

	y_t	u_t	y_{t-1}	u_{t-1}	1
y_t	1	$\cdot$	$\cdot$	$\cdot$	$-s_t$
u_t	$\cdot$	$\omega + \lambda$	$\cdot$	$-\lambda$	$\cdot$
y_{t-1}	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
u_{t-1}	$\cdot$	$-\lambda$	$\cdot$	λ	$\cdot$
1	$-s_t$	$\cdot$	$\cdot$	$\cdot$	s_t^2

Completion to square

Square

$$(x+m)^2 = x^2 + 2mx + m^2$$

Completion to the square

$$x^2 + 2mx + n^2 = x^2 + 2mx + m^2 - m^2 + n^2 = (x+m)^2 + n^2 - m^2$$

In vectors

$$x'Ax + 2x'Bm + m'Cm = x'Ax + \underbrace{2x'AA^{-1}Bm}_{2x'Bm} + \underbrace{m'B'A^{-1}\overbrace{AA^{-1}}^{I}Bm}_{\text{square in } m}$$

$$-\underbrace{m'B'A^{-1}Bm}_{\text{square in } m} + m'Cm =$$

$$\underbrace{(x-m)'A(x-m)}_{\text{full square}} + \underbrace{m'Cm - m'B'A^{-1}Bm}_{\text{reminder}}$$