

Regression model

[Scalar regression model](#)

[Multivariate regression model](#)

[Theoretical derivation of estimation](#)

Regression model is the most frequently used model for continuous target variable and most of the explanatory variable continuous, too.

Its conditional pdf is given by the equation

$$y_t = \psi_t' \theta + e_t$$

where y_t is the target variable at time t and ψ_t consists of values of those variables that influence y_t and Θ are model parameters, i.e. of regression coefficient θ and variance of Gaussian noise e_t (its expectation is zero).

Usually, the regression vector ψ_t contains

- only constant - descriptive model

$$y_t = k + e_t$$

where the model pdf is a fixed Gaussian bell function with the top at the value k .

- explanatory variables - explanatory model

$$y_t = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n + e_t$$

where v_i are explanatory variables that explain behavior of y_t . The model pdf is a Gaussian bell function with the top given by $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$.

- control and delayed outputs - predictive model

$$y_t = b_0 u_t + a_1 y_{t-1} + b_2 u_{t-1} + \cdots + a_n y_{t-n} + b_n u_{t-n} + k + e_t$$

where u is the control variable. The model pdf is a moving Gaussian bell function following the transient effect of the system dynamics. Its top lies at the point $b_0 u_t + a_1 y_{t-1} + b_2 u_{t-1} + \cdots + a_n y_{t-n} + b_n u_{t-n} + k$ and for u being a unit step its steady state value is $y_\infty = \frac{\sum b_i + k}{1 - \sum a_i}$.

Estimation

Obeys the Bays rule

$$f(\Theta|d(t)) \propto f(y_t|\psi_t, \Theta) f(\Theta|d(t-1))$$

where $\Theta = \{\theta, r\}$ is a collection of parameters (θ are regression coefficients and r is variance of noise), $d(t)$ is a set of all data measured up to t (including prior data) and

- $f(\Theta|d(t))$ is posterior pdf
- $f(\Theta|d(t-1))$ is prior pdf (posterior from the last step)
- $f(y_t|\psi_t, \Theta)$ is model.

A practical form of the Bayes rule is obtained after parametrization of the distributions using some known distribution. Then this functional recursion becomes an algebraic one which is feasible for computation.