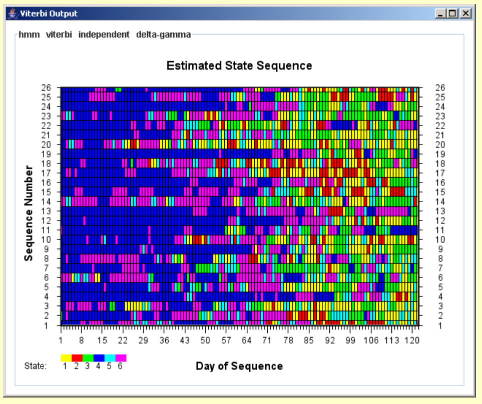
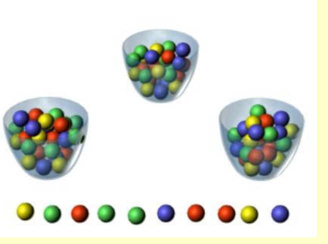
Hidden Markov Model

- The simplest dynamic Bayesian nework
- Application in speech recognition, handwriting, gesture recognition…



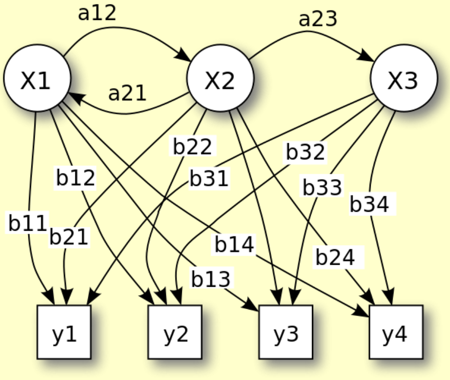


Hidden Markov Model

- In a room that is not visible to an observer there is a genie. The room contains urns X1, X2, X3, … each of which contains a known mix of balls, each ball labeled y1, y2, y3, … . The genie chooses an urn in that room and randomly draws a ball from that urn.

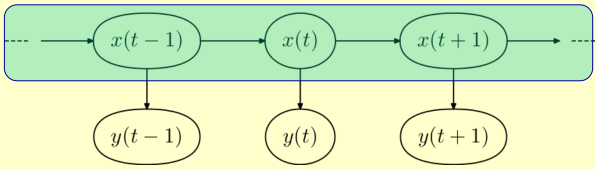
Observer can observe the sequence of the balls but not the sequence of urns from which they were drawn.

States = {X1, X2, X3}
Observations = {y1,y2,y3,y4}



Transition probabilities **aij** = P(Xi -> Xj)
Emission probabilities **bij** =P(Xi -> yj)

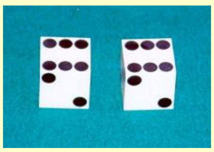
Hidden Markov Model



Hidden state sequence *x*(t-1), *x*(t), *x*(t+1). Markov process which is hidden behind a green screen is determined by the current state and transition matrix **P**. We are only able to observe the *y*(t) which are related to the hidden states by emission matrix **E**.

States = {x1, x2, x3}
Observations = {y1,y2,y3,y4}
Transition probabilities $\mathbf{P}_{ij}$ **=** P(Xi -> Xj)
Emission probabilities $\mathbf{E}_{ij}$ =P(Xi -> yj)

Example: The dishonest casino



A casino has two dice:

- Fair die
  P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6
- Loaded die
  P(1) = P(2) = P(3) = P(4) = P(5) = 1/10
  P(6) = 1/2

Casino player switches between fair and loaded die with probability 1/20 at each turn

Game:

1. You bet $1
2. You roll (always with a fair die)
3. Casino player rolls (maybe with fair die, maybe with loaded die)
4. Highest number wins $2

Question # 1 – Decoding

GIVEN

A sequence of rolls by the casino player

O = **455264621461461361366616646616366163661636165156151151461235623**

FAIR | LOADED | FAIR

QUESTION

What portion of the sequence was generated with the fair die, and what portion with the loaded die?

Given $HMM = (P, E, p0)$ and observation O.

Find optimal state sequence X for the underlying Markov process.

Question # 2 – Evaluation

GIVEN

A sequence of rolls by the casino player

O = **4552646214614613613666166466163661636616361651561511514612356234**

Prob = 1.3×10^{-35}

QUESTION

How likely is this sequence, given our model of how the casino works?

$$HMM = (P, E, p0)$$

$$P(O / HMM) = ?$$

Question # 3 – Training

GIVEN

A sequence of rolls by the casino player

45526462146146136136661664661636616366163616515615115146123

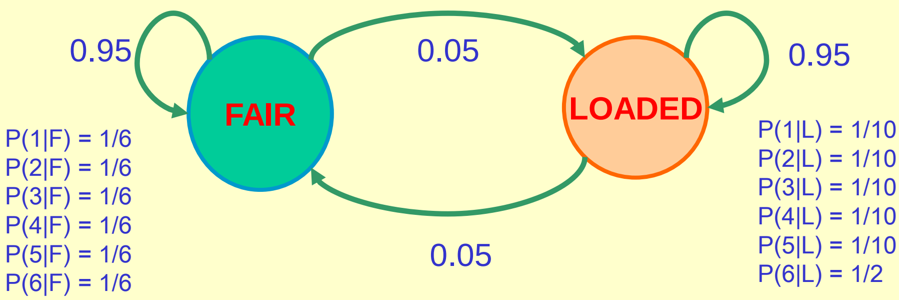
Prob(6) = 64%

QUESTION

How “loaded” is the loaded die? How “fair” is the fair die? How often does the casino player change from fair to loaded, and back?

Given O and number states.

$$HMM = (P, E, p0) = ?$$



$$P = \begin{matrix} F \\ L \end{matrix} \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix}$$

$$E = \begin{matrix} \\ F \\ L \end{matrix} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \left(\frac{1}{6} \right. & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \left. \frac{1}{6} \right) \\ .1 & .1 & .1 & .1 & .1 & .5 \end{matrix}$$

Question # 2 – Evaluation

Initial distribution p0 = (1,0) – we start with fair dice

Consider observation sequence O = **4552**

and state sequence X ={F,F,L,L}

$$P\left(4552 \cap FFLL\right) =$$
$$= p0(F) * E(F4) * P(FF) * E(F5) * P(FL) * E(L5) * P(LL) * E(L2) =$$
$$= 1 \cdot \frac{1}{6} \cdot 0.95 \cdot \frac{1}{6} \cdot 0.05 \frac{1}{10} 0.95 \cdot \frac{1}{10} = 1.25 \cdot 10^{-5}$$
$$P\left(4552 \cap FFFL\right)$$

...

all16 variation

By summing over all possible state sequence we obtain P(O1, O2, O3, O4)

Question # 2 – Evaluation forward algorithm

Given: HMM: p0 - Initial distribution

P - Transition matrix (nxn) P(i,j) = P(state *i* -> state *j*)

E – Emission matrix (nxm) E(i,j) = P(state *i* -> O*j*)

sequence of observation O=(O1, O2, …, OT)

Find P(O1, O2, …, OT)

For t =1,2, …T define vector $\alpha(t)$, where $\alpha_i(t)$ = P(O1,…Ot, i state)

$$\alpha(1) = \left(p0_1 \cdot E(1,O1), p0_2 \cdot E(2,O1), ..., p0_n \cdot E(n,O1)\right) = p0.* E(:,O1)$$
$$\alpha_j(t) = E(j,Ot) \cdot \sum_{i=1..n} \alpha_i(t-1) * P(i,j)$$
$$\alpha(t) = E(:,Ot).*\alpha(t-1) \cdot P$$
$$P\left(O1,O2,...OT\right) = \sum_{i=1..n} \alpha_i(T)$$

Question # 2 – Evaluation forward algorithm

$$\alpha(1) = \left(p0_1 \cdot E(1,O1), p0_2 \cdot E(2,O1), \ldots, p0_n \cdot E(n,O1) \right) = p0.*E(:,O1)$$
$$\alpha(t) = E(:,Ot).*\alpha(t-1) \cdot P$$

$$P = \begin{matrix} F \\ L \end{matrix} \begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix}$$

```
p0 = [1,0];
observ = [4,5,1,2,6,6,6];
alpha(:,1)=p0.*E(:,observ(1))

alpha(:,2)=alpha(:,1)'*P.*E(:,observ(2))'
```

$$E = \begin{matrix} \\ F \\ L \end{matrix} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \left(\frac{1}{6} \right. & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \left. \frac{1}{6} \right) \\ .1 & .1 & .1 & .1 & .1 & .5 \end{matrix}$$

```
sum(alpha(:,T))
```

$$\alpha = \begin{pmatrix} \frac{1}{6} & . & . & . & . & . & \alpha_F(T) \\ 0 & . & . & . & . & . & \alpha_L(T) \end{pmatrix}$$

Question # 1 – Decoding

Initial state p0 = (1,0) – we start with fair dice

Given $HMM = (P, E, p0)$ and observation O.

Find optimal state sequence X for the underlying Markov process.

```
P=[0.95,0.05;0.05,0.95];
E=[1/6,1/6,1/6,1/6,1/6,1/6;1/10,1/10,1/10,1/1
0,1/10,1/2];
observ=[4,5,1,2,6,6,6];
pStates = hmmdecode(observ,P,E); % P(X/O)
estStates= hmmviterbi(observ,P,E);
```

	F	0,95	0,89	0,79	0,62	0,33	0,24	0,22
pStates =	L	0,04	0,11	0,21	0,38	0,67	0,76	0,78
estStates =		1	1	1	1	2	2	2

Hidden Markov Model - example

- States ={'OK';'defect';'serious defect'}
- Observations ={1,2,3,4}

$$P = \begin{pmatrix} 0.9 & 0.1 & 0.1 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Transition probabilities Pij = P(Xi -> Xj)
b) Emission probabilities Eij = P(Xi -> yj)

$$E = \begin{matrix} OK \\ defect \\ ser.def \end{matrix} \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.1 & 0.1 & 0.8 \end{pmatrix}$$

```
%generovani skryte posloupnosti states a pozorovane
posloupnosti observ
[observ,states] = hmmgenerate(100,P,E)

[estimateP,estimateE] = hmmestimate(observ,states);
```

HiddenMarkov.m