

1) Exercise: Calculate the following determinants

$$a) \begin{vmatrix} 2 & 5 \\ -2 & 3 \end{vmatrix} = 6 + 10 = 16$$

$$b) \begin{vmatrix} 1 & -3 \\ -4 & 6 \end{vmatrix} = 6 - 12 = -6$$

$$c) \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$$

Try to solve d, e.

$$f) \begin{vmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{vmatrix} = \sin^2 x + \cos^2 x = 1$$

$$g) \begin{vmatrix} \sin x & -\sin y \\ \cos x & \cos y \end{vmatrix} = \sin x \cos y + \sin y \cos x = \sin(x+y)$$

Try to solve h, i, j.

2) Exercise: Calculate the following determinants

$$a) \begin{vmatrix} 5 & -2 & 1 \\ -5 & 3 & 4 \\ 2 & 1 & 3 \\ 3 & -2 & 1 \\ -5 & 3 & 4 \end{vmatrix} = 24 - 5 - 16 - (6 + 12 + 30) = -42$$

$$b) \begin{vmatrix} 4 & 10 & 1 \\ 0 & 2 & 0 \\ 1 & -3 & 4 \\ -4 & 10 & 1 \\ 0 & 2 & 0 \end{vmatrix} = 56 + 0 + 0 - (2 + 0 + 0) = 54$$

$$c) \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{vmatrix} = 1+0+4 - (0+3+0) = 2$$

$$d) \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0+1+1 - (0+0+0) = 2$$

Try to solve e, f, g, h, i, j .

$$b) \begin{vmatrix} 1 & 0 & f \\ u & 1 & k \\ 0 & 1 & k \end{vmatrix} = k+0+uf - (0+k+0) = uf$$

$$l) \begin{vmatrix} 0 & a & a \\ a & 0 & a \\ a & a & 0 \end{vmatrix} = 0+a^3+a^3 - (0+0+0) = 2a^3$$

Try to solve m, n, o .

3) Exercise: Calculate the following determinants

$$a) \begin{vmatrix} 1 & 2 & 3 & 4 \\ -5 & 2 & -1 & 1 \\ -6 & 5 & 2 & 1 \\ 3 & -1 & 1 & 0 \end{vmatrix} \begin{matrix} 5 \\ 6 \\ -3 \\ + \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 12 & 14 & 21 \\ 0 & 14 & 20 & 25 \\ 0 & -7 & -8 & -12 \end{vmatrix} = (-1)^{1+1} \cdot 1 \begin{vmatrix} 12 & 14 & 21 \\ 14 & 20 & 25 \\ -7 & -8 & -12 \end{vmatrix}$$

$$= -2380 - 2450 - 2856 - (-2940 - 2400 - 2856) =$$

$$= 10$$

$$b) \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{matrix} -1 \\ + \\ + \\ + \end{matrix} = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} = (-1)^{1+1} \cdot 1 \begin{vmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= 0 - 1 - 1 - (0 + 0 + 1) = -3$$

Try to solve c, d, e, f, g, h, i .

$$\begin{aligned}
 1) \quad & \begin{vmatrix} a & 1 & 1 & 1 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 1 \\ d & 1 & 1 & 0 \end{vmatrix} \begin{matrix} (-1) \\ + \\ + \\ + \end{matrix} = \begin{vmatrix} a & 1 & 1 & 1 \\ b-a & -1 & 0 & 0 \\ c-a & 0 & -1 & 0 \\ d & 1 & 1 & 0 \end{vmatrix} = (-1)^{1+4} \cdot 1 \begin{vmatrix} b-a & -1 & 0 \\ c-a & 0 & -1 \\ d & 1 & 1 \end{vmatrix} \\
 & = -[0 + d + 0 - (0 - b + a - c + a)] = \\
 & = -d - b - c + 2a
 \end{aligned}$$

Try to solve b, c, m, n .

4) Exercise: Solve the equation

$$a) \quad \begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0$$

$$-4x^2 + 0 + 2x - (0 + x^2 + 12x) = 0$$

$$-5x^2 - 10x = 0$$

$$-5x(x+2) = 0 \quad \underline{x=0, x=-2}$$

Try to calculate b, c, d .

5) Exercise: Find the general equation of the straight-line AB and calculate the area of the triangle ABC

$$a) \quad \underline{A = [-1, 5], B = [2, -6], C = [4, 0]}$$

$$\overleftrightarrow{AB} \quad \begin{vmatrix} -1 & 5 & 1 \\ 2 & -6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\begin{aligned}
 6 + 5x + 2y - (-6x - y + 10) &= 0 \\
 \underline{11x + 3y - 4} &= 0
 \end{aligned}$$

$$\begin{aligned}
 S_{ABC} &= \frac{1}{2} \left| \begin{vmatrix} -1 & 5 & 1 \\ 2 & -6 & 1 \\ 4 & 0 & 1 \end{vmatrix} \right| = \frac{1}{2} |6 + 20 + 0 - (-24 + 0 + 10)| = \frac{1}{2} |26 + 14| \\
 &= \underline{\underline{20}}
 \end{aligned}$$

Try to solve b, c.

6) Exercise: Find the general equation of the plane ABC and calculate the volume of the tetrahedron ABCD

a) $A = [3, 0, 4], B = [-1, -1, 7], C = [0, -2, -3], D = [6, 5, 4]$.

$$\begin{aligned}
 \text{ABC} \quad 0 &= \begin{vmatrix} 3 & 0 & 4 & 1 \\ -1 & -1 & 7 & 1 \\ 0 & -2 & -3 & 1 \\ x & y & z & 1 \end{vmatrix} \begin{matrix} -1 \\ + \\ + \\ + \end{matrix} = \begin{vmatrix} 3 & 0 & 4 & 1 \\ -1 & -1 & 3 & 0 \\ -3 & -2 & -7 & 0 \\ x-3 & y & z-4 & 0 \end{vmatrix} = (-1)^{1+4} \cdot 1 \begin{vmatrix} -4 & -1 & 3 \\ -3 & -2 & -7 \\ x-3 & y & z-4 \end{vmatrix} \\
 &= - [8(z-4) - 9y + 7(x-3) + 6(x-3) - 28y - 3(z-4)] \\
 &= - 5(z-4) + 37y - 13(x-3) = -5z + 20 + 37y - 13x + 39 \\
 &= -13x + 37y - 5z + 59
 \end{aligned}$$

$S_{ABC} \quad -13x + 37y - 5z + 59 = 0$

$$\begin{aligned}
 V_{ABCD} &= \frac{1}{6} \begin{vmatrix} 3 & 0 & 4 & 1 \\ -1 & -1 & 7 & 1 \\ 0 & -2 & -3 & 1 \\ 6 & 5 & 4 & 1 \end{vmatrix} \begin{matrix} -1 \\ + \\ + \\ + \end{matrix} = \frac{1}{6} \begin{vmatrix} 3 & 0 & 4 & 1 \\ -1 & -1 & 3 & 0 \\ -3 & -2 & -7 & 0 \\ 3 & 5 & 0 & 0 \end{vmatrix} = \frac{1}{6} (-1)^{4+1} \cdot 1 \begin{vmatrix} -4 & -13 \\ -3 & -2-7 \\ 3 & 5 & 0 \end{vmatrix} \\
 &= \frac{1}{6} | 0 + 21 - 45 + 18 - 0 - 140 | = \frac{1}{6} |-146| = \frac{146}{6} = \frac{73}{3}
 \end{aligned}$$

Try to solve b.