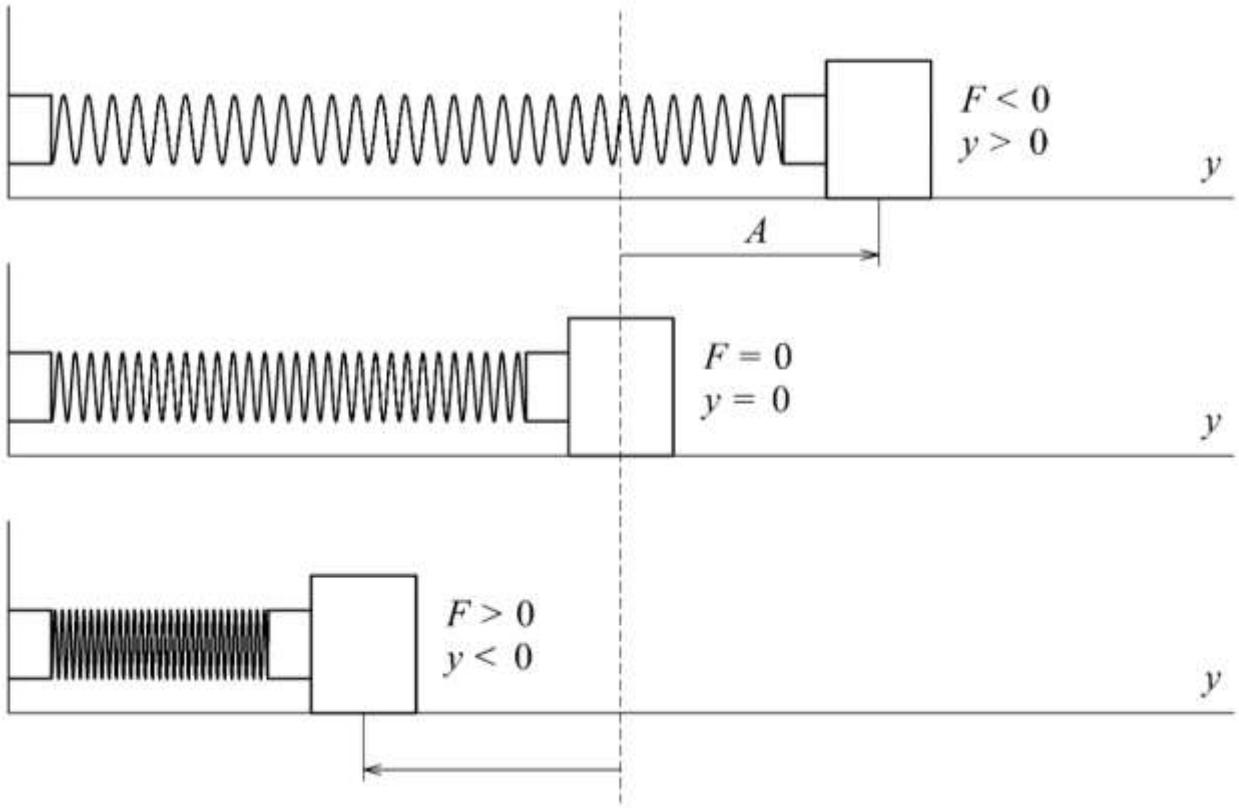
Harmonický oscilátor

Harmonický oscilátor netlumený



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

$$F = m\frac{d^2 y}{dt^2}$$

$$F = -ky$$

Harmonický oscilátor netlumený

$$m\frac{d^2y}{dt^2} = -ky$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0 \qquad y = C\,\mathrm{e}^{\lambda t}$$

$$\lambda^2 C\,\mathrm{e}^{\lambda t} + \frac{k}{m}C\,\mathrm{e}^{\lambda t} = 0 \qquad \omega^2 = \frac{k}{m}$$

$$\lambda^2 + \omega^2 = 0$$

$$\lambda = \pm i\omega$$

$$y = C_1\,\mathrm{e}^{i\omega t} + C_2\,\mathrm{e}^{-i\omega t} = C_1\left(\cos\omega t + i\sin\omega t\right) + C_2\left(\cos\omega t - i\sin\omega t\right) =$$

$$= \left(C_1 + C_2\right)\cos\omega t + i\left(C_1 - C_2\right)\sin\omega t = K_1\cos\omega t + K_2\sin\omega t$$

$$K_1 = C_1 + C_2 \qquad K_2 = i\left(C_1 - C_2\right)$$

$$y = K_1 \cos \omega t + K_2 \sin \omega t$$

$$K_1 = A \sin \varphi_0 \qquad A^2 = K_1^2 + K_2^2$$

$$K_2 = A \cos \varphi_0 \qquad \operatorname{tg} \varphi_0 = \frac{K_1}{K_2}$$

$$y = A \sin\left(\omega t + \varphi_0\right)$$

Harmonický oscilátor netlumený

$$m\frac{d^2y}{dt^2} = -ky$$

$$y = A\sin\left(\omega t + \varphi_0\right) \qquad \omega^2 = \frac{k}{m}$$

$$v = \frac{dy}{dt} = A\omega\cos\left(\omega t + \varphi_0\right)$$

$$a = \frac{dv}{dt} = -A\omega^2\sin\left(\omega t + \varphi_0\right) = -\omega^2 A\sin\left(\omega t + \varphi_0\right) = -\omega^2 y$$

práce na pružině $$A = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_0^y F_y \cdot dy = -\int_0^y ky \cdot dy = -\frac{1}{2}ky^2$$

potenciální energie hmotného bodu konajícího kmitavý harmonický pohyb v závislosti na okamžité výchylce y

$$\Delta W_p = A' = -A$$

$$W_p = \frac{1}{2}ky^2$$

kinetická energie hmotného bodu $$W_k = \frac{1}{2}mv^2$$

$$W = W_k + W_p = \frac{1}{2}mv^2 + \frac{1}{2}ky^2$$

$$W = \frac{1}{2}m\left[A\omega\cos\left(\omega t + \varphi_0\right)\right]^2 + \frac{1}{2}k\left[A\sin\left(\omega t + \varphi_0\right)\right]^2 =$$

$$= \frac{1}{2}m\left[A^2\frac{k}{m}\cos^2\left(\omega t + \varphi_0\right)\right] + \frac{1}{2}k\left[A^2\sin^2\left(\omega t + \varphi_0\right)\right] = \frac{1}{2}kA^2$$

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Harmonický oscilátor tlumený

$$m\frac{d^2y}{dt^2} = -ky \; - B\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + \frac{B}{m}\frac{dy}{dt} + \frac{k}{m}y = 0 \qquad \frac{k}{m} = \omega_0^2 \qquad \frac{B}{m} = 2b$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega_0^2 y = 0 \qquad y = C\,\mathrm{e}^{\lambda t}$$

$$\lambda^2 + 2b\lambda + \omega_0^2 = 0$$

$$\lambda_{1/2} = -b \pm \sqrt{b^2 - \omega_0^2}$$

$b = \omega_0$

$$y = C_1 \mathrm{e}^{-bt} + C_2 t\, \mathrm{e}^{-bt} = \mathrm{e}^{-bt}\left(C_1 + C_2 t\right)$$

$b > \omega_0$

$$D = \sqrt{b^2 - \omega_0^2}$$

$$y = C_1 \mathrm{e}^{(-b+D)t} + C_2 \mathrm{e}^{(-b-D)t}$$ aperiodický pohyb

$b < \omega_0$

$$\lambda_1 = -b + i\sqrt{\omega_0^2 - b^2}$$

$$\lambda_2 = -b - i\sqrt{\omega_0^2 - b^2}$$

$$\omega = \sqrt{\omega_0^2 - b^2}$$

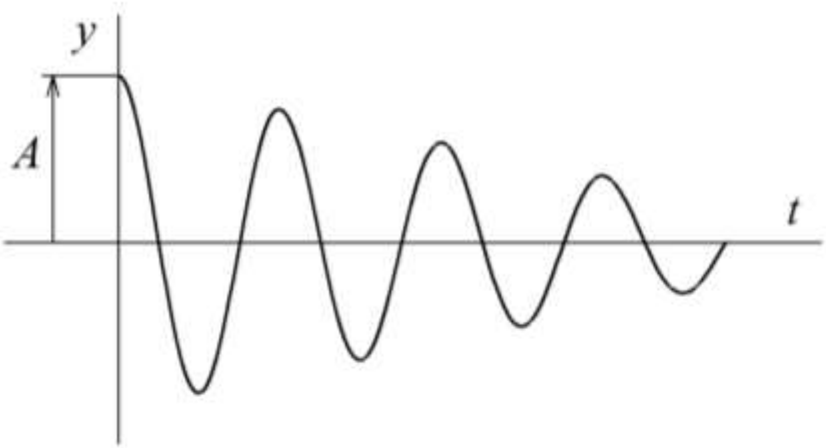
$$y = \mathrm{e}^{-bt}\left(C_1 \mathrm{e}^{i\omega t} + C_2 \mathrm{e}^{-i\omega t}\right) = A \mathrm{e}^{-bt} \sin\left(\omega t + \varphi_0\right)$$

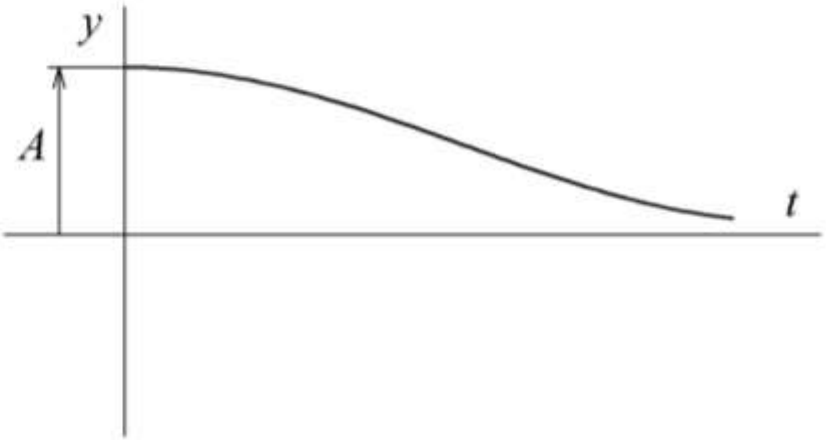
Harmonický oscilátor tlumený

$$m\frac{d^2y}{dt^2} = -ky - B\frac{dy}{dt}$$

$$y = Ae^{-bt}\sin\left(\omega t + \varphi_0\right)$$

$$\omega = \sqrt{\omega_0^2 - b^2}$$





Nucené kmitání

$$m\frac{d^2y}{dt^2} = -ky - B\frac{dy}{dt} + F_0 \sin\Omega t$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega_0^2 y = \frac{F_0}{m}\sin\Omega t$$

$$y = A\sin\left(\Omega t + \varphi_0\right)$$

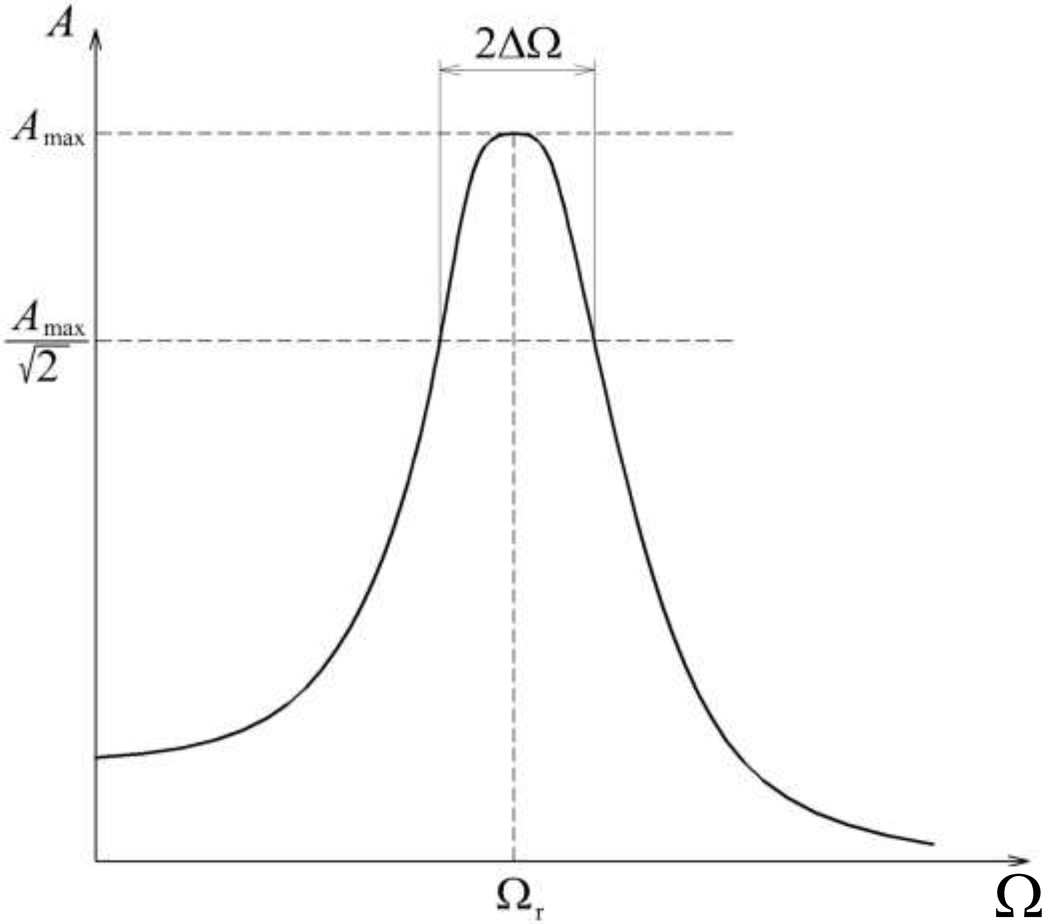
$$A = \frac{F_0}{m\left[\left(\omega_0^2 - \Omega^2\right)^2 + 4b^2\Omega^2\right]^{\frac{1}{2}}} \qquad \mathrm{tg}\,\varphi_0 = -\frac{2b\Omega}{\omega_0^2 - \Omega^2}$$

Rezonance

$$A = \frac{F_0}{m\left[\left(\omega_0^2 - \Omega^2\right)^2 + 4b^2\Omega^2\right]^{\frac{1}{2}}}$$

$$\Omega_r = \left(\omega_0^2 - 2b^2\right)^{\frac{1}{2}}$$

$$b \doteq 0 \Rightarrow \Omega_r \doteq \omega_0$$



https://www.youtube.com/watch?v=3mclp9QmCGs
https://www.youtube.com/watch?v=JUVgouE_sg0