Soustava hmotných bodů

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$$\frac{d\vec{p}_i}{dt} = \vec{F}_i = \vec{F}_{i\,\mathrm{ext}} + \vec{F}_{i\,\mathrm{int}}$$

$$\vec{F}_{ik\,\mathrm{int}} = -\vec{F}_{ki\,\mathrm{int}}$$

$$\sum_i \frac{d\vec{p}_i}{dt} = \frac{d}{dt}\sum_i (m_i\vec{v}_i) = \frac{d\vec{p}}{dt} = \sum_i \vec{F}_{i\,\mathrm{ext}} = \vec{F}$$

izolovaná soustava $\vec{F} = 0$

střed hmotnosti soustavy $\vec{r}_s = \frac{1}{m}\sum_i m_i\vec{r}_i$

$$\vec{v}_s = \frac{1}{m}\sum_i m_i\vec{v}_i$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p} = \sum_i m_i\vec{v}_i = m\vec{v}_s$$

$$\vec{F} = \sum_i \vec{F}_i = 0 \Rightarrow \vec{p} = \overrightarrow{\mathrm{konst}}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}_s$$

zákon zachování hybnosti

https://www.matfyz.cz/clanky/fyzikalni-pokus-razostroj

Srážka dvou částic

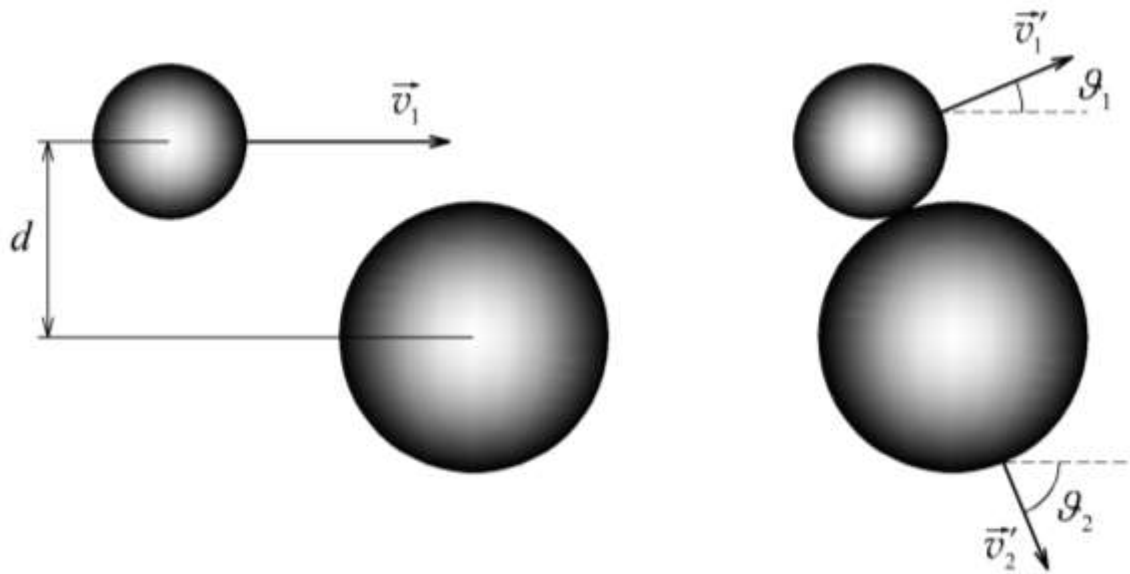
https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_en.html

izolovaná soustava 2 částic

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

srážka pružná
nepružná



ZZE

$$m_1 v_1 = m_1 v_1' \cos\vartheta_1 + m_2 v_2' \cos\vartheta_2$$

$$0 = m_1 v_1' \sin\vartheta_1 - m_2 v_2' \sin\vartheta_2$$

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

Laboratorní soustava

$$\vec{v}_s = \frac{m_1 \vec{v}_1}{m_1 + m_2}$$

Těžišťová soustava

$$W_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

$$\vec{p}_1 = -\vec{p}_2$$

$$W_k = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = p_1^2\left(\frac{1}{2m_1} + \frac{1}{2m_2}\right)$$

$$\vec{p}_1' = -\vec{p}_2'$$

$$W_k = p_1'^2\left(\frac{1}{2m_1} + \frac{1}{2m_2}\right)$$

pružná srážka $\quad p_1^2 = p_1'^2$

$$\vec{p}_1 = \vec{p}_1'$$

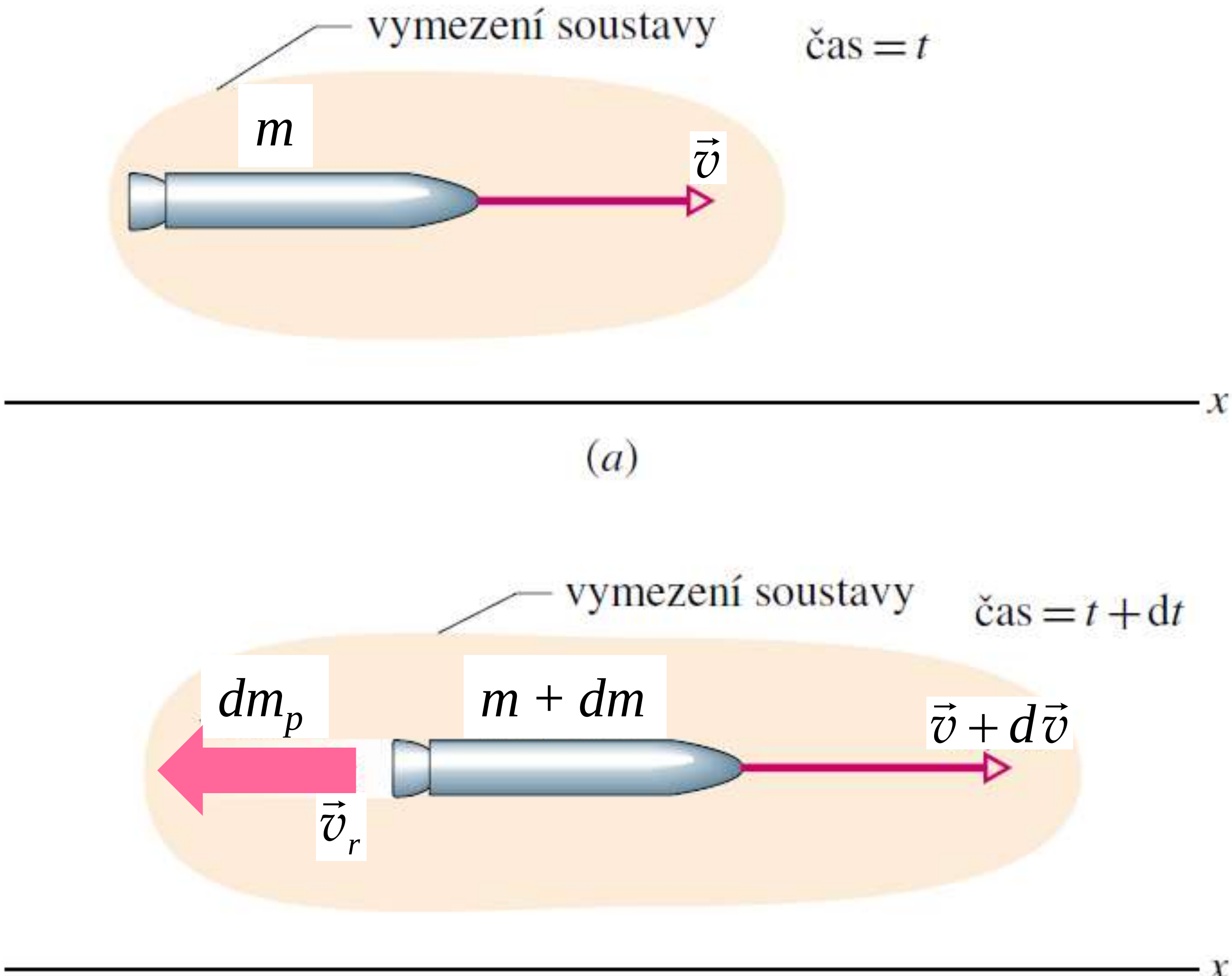
$$\vec{p}_2 = \vec{p}_2'$$

$$\vec{p}_1 = -\vec{p}_1' \qquad \left(\vec{v}_1 = -\vec{v}_1'\right)$$

$$\vec{p}_2 = -\vec{p}_2' \qquad \left(\vec{v}_2 = -\vec{v}_2'\right)$$

Princip pohybu rakety

Halliday, Resnik, Walker:
Fyzika, Prometheus, 2003



Princip pohybu rakety

$$\vec{p}(t) = m\vec{v}(t)$$

$$dm_p = -dm$$

$$\vec{v}_p = \vec{v} + \vec{v}_r$$

$$\vec{p}(t+dt) = (m+dm)(\vec{v}+d\vec{v}) + dm_p\vec{v}_p$$

$$d\vec{p} = \vec{p}(t+dt) - \vec{p}(t) = m\vec{v} + md\vec{v} + \vec{v}dm + dm\,d\vec{v} + dm_p\vec{v}_p - m\vec{v} =$$

$$= md\vec{v} + \vec{v}dm + dm_p\vec{v}_p = md\vec{v} + \vec{v}dm - dm\,\vec{v}_p =$$

$$= md\vec{v} + (\vec{v} - \vec{v}_p)dm$$

$$d\vec{p} = md\vec{v} - \vec{v}_r dm$$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} - \vec{v}_r\frac{dm}{dt} = \vec{F}$$

$$m\frac{d\vec{v}}{dt} = \vec{F} + \vec{v}_r\frac{dm}{dt}$$ rovnice Měščerského

$\vec{F} = 0$ 1. Ciolkovského úloha $\quad \vec{F} = 0 \Rightarrow m\frac{d\vec{v}}{dt} = \vec{v}_r\frac{dm}{dt}$

$$m\frac{d\vec{v}}{dt} = \vec{v}_r \frac{dm}{dt} \qquad \frac{dm_p}{dt} = \mu \qquad \frac{dm}{dt} = -\mu \qquad m = m_0 - \mu t$$

$$\left(m_0 - \mu t\right)\frac{d\vec{v}}{dt} = -\mu\vec{v}_r$$

$$d\vec{v} = -\vec{v}_r \frac{\mu}{m_0 - \mu t} dt$$

$$\vec{v} = \int -\vec{v}_r \frac{\mu}{m_0 - \mu t} dt \; = \vec{v}_r \int \frac{-\mu}{m_0 - \mu t} dt = \vec{v}_r \ln\left(m_0 - \mu t\right) + C$$

$$\vec{v}_0 = \vec{v}_r \ln m_0 + C \Rightarrow C = \vec{v}_0 - \vec{v}_r \ln m_0$$

$$\vec{v} = \vec{v}_r \ln\left(m_0 - \mu t\right) + \vec{v}_0 - \vec{v}_r \ln m_0 \; = -\vec{v}_r \left[\ln m_0 - \ln\left(m_0 - \mu t\right)\right] + \vec{v}_0$$

$$\vec{v} = -\vec{v}_r \ln\frac{m_0}{m} + \vec{v}_0$$