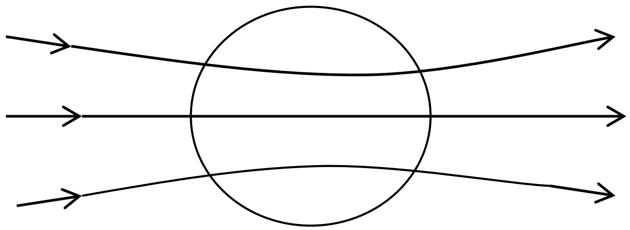
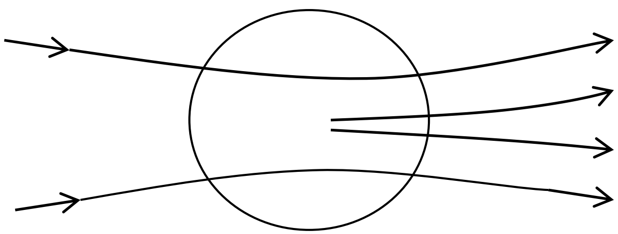
# Mechanika tekutin

$\operatorname{div} \vec{v} = 0$



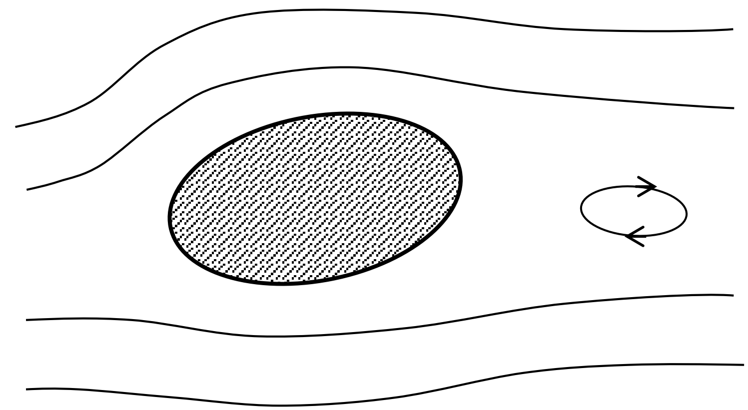
$\operatorname{div} \vec{v} \neq 0$



$\operatorname{div} \vec{v} \neq 0$ proudění zřídlové

$\operatorname{rot} \vec{v} = \vec{\nabla} \times \vec{v}$

$\operatorname{rot} \vec{v} \neq 0$

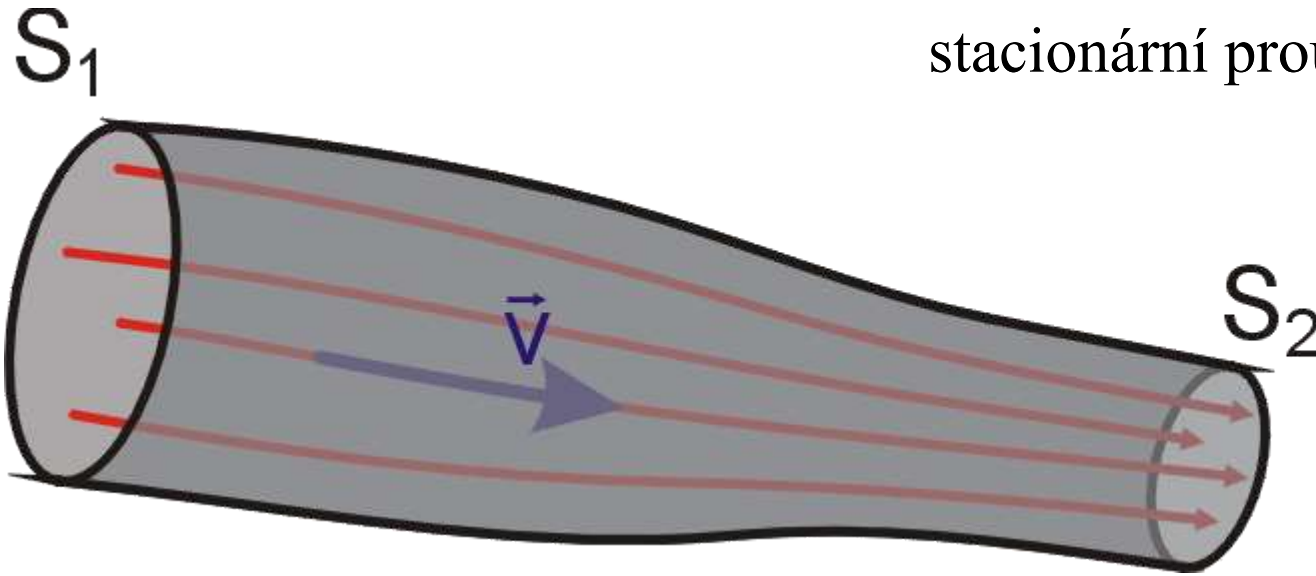


$\operatorname{rot} \vec{v} \neq 0$ proudění vírové

$$\vec{v} = \vec{v}_T + \left(\vec{\omega} \times \vec{r}\right)$$

$$\operatorname{rot} \vec{v} = \operatorname{rot} \vec{v}_T + \operatorname{rot}\left(\vec{\omega} \times \vec{r}\right)$$

$$\operatorname{rot} \vec{v} = 2\vec{\omega}$$




stacionární proudění

http://physics.mff.cuni.cz/kfpp/skripta/kurz_fyziky_pro_DS/www/fyzika.html

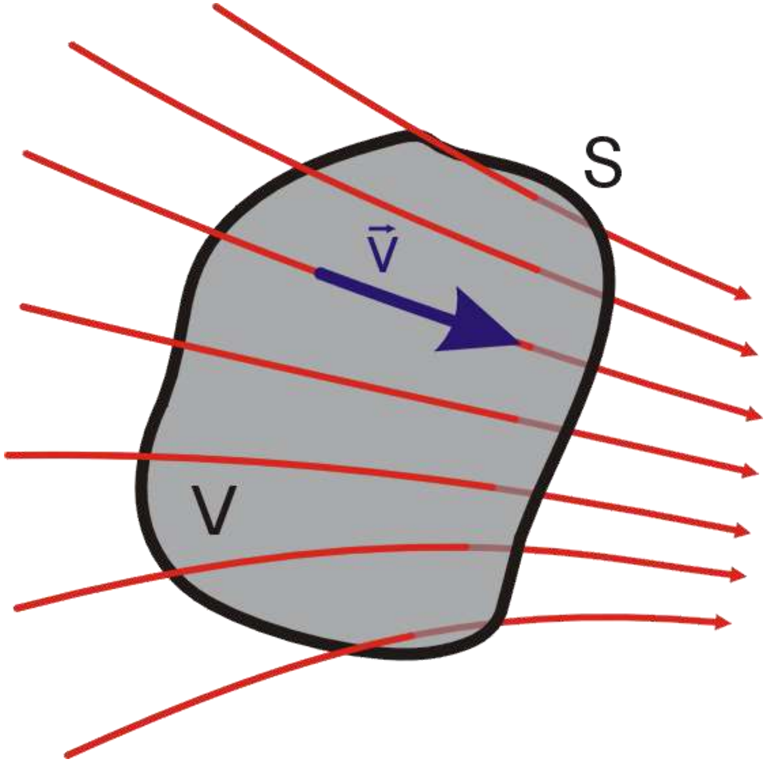
$$\Delta m_1 = \rho_1 S_1 v_1 \Delta t$$

$$\Delta m_2 = \rho_2 S_2 v_2 \Delta t$$

$$\frac{\Delta m}{\Delta t} = \rho S v$$

$$\rho_1 S_1 v_1 = \rho_2 S_2 v_2$$ rovnice kontinuity

$$S_1 v_1 = S_2 v_2$$ kapalina




$$\frac{\partial m'}{\partial t} = \oiint_S \rho \vec{v} \cdot d\vec{S} \qquad \frac{\partial m'}{\partial t} = -\frac{\partial m}{\partial t}$$

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} \iiint_V \rho \, dV$$

$$\oiint_S \rho \vec{v} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint_V \rho \, dV = -\iiint_V \frac{\partial \rho}{\partial t} dV$$

$$\oiint_S \rho \vec{v} \cdot d\vec{S} = \iiint_V \operatorname{div}(\rho \vec{v}) \, dV$$

$$\iiint_V \operatorname{div}(\rho \vec{v}) \, dV = -\iiint_V \frac{\partial \rho}{\partial t} dV$$

$$\iiint_V \left[ \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) \right] dV = 0$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0$$

rovnice kontinuity

$$-\frac{d\rho}{dt} = \rho \operatorname{div} \vec{v}$$

http://physics.mff.cuni.cz/kfpp/skripta/kurz_fyziky_pro_DS/www/fyzika.html

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\left(\rho \vec{v}\right) = 0$$

nestlačitelná tekutina $\frac{\partial \rho}{\partial t} = 0$ $\mathrm{grad}\,\rho = 0$

$\mathrm{div}\,\vec{v} = 0$ nezřídlové proudění

# Bilance hybnosti a energie

$$\Delta m\vec{a} = \Delta\vec{F}_0 + \Delta\vec{P}_t + \Delta\vec{P}_V$$

$$\rho\vec{a} = \rho\vec{K} - \text{grad}\, p + \vec{f}_V$$ pohybová rovnice pro vazkou tekutinu

$$\rho a_1 = \rho K_1 - \frac{\partial p}{\partial x_1} + f_{V1}$$

ideální tekutina (bez tření)

$$\rho\vec{a} = \rho\vec{K} - \text{grad}\, p$$ **Eulerova pohybová rovnice**

$$\rho a_1 = \rho K_1 - \frac{\partial p}{\partial x_1}$$

$$\rho\vec{a} = -\rho\, \text{grad}\, U - \text{grad}\, p$$

# Pascalův zákon

$$\rho\vec{a} = \rho\vec{K} - \mathrm{grad}\, p$$

$$0 = \rho\vec{K} - \mathrm{grad}\, p \qquad \rho\vec{K} = 0$$

$$0 = -\mathrm{grad}\, p \quad \Rightarrow \quad \boxed{p(\vec{r}) = \mathrm{konst}}$$

# Hydrostatický tlak

$$\rho\vec{a} = \rho\vec{K} - \operatorname{grad} p$$

$$0 = \rho\vec{K} - \operatorname{grad} p$$

$$0 = \rho\vec{g} - \operatorname{grad} p$$

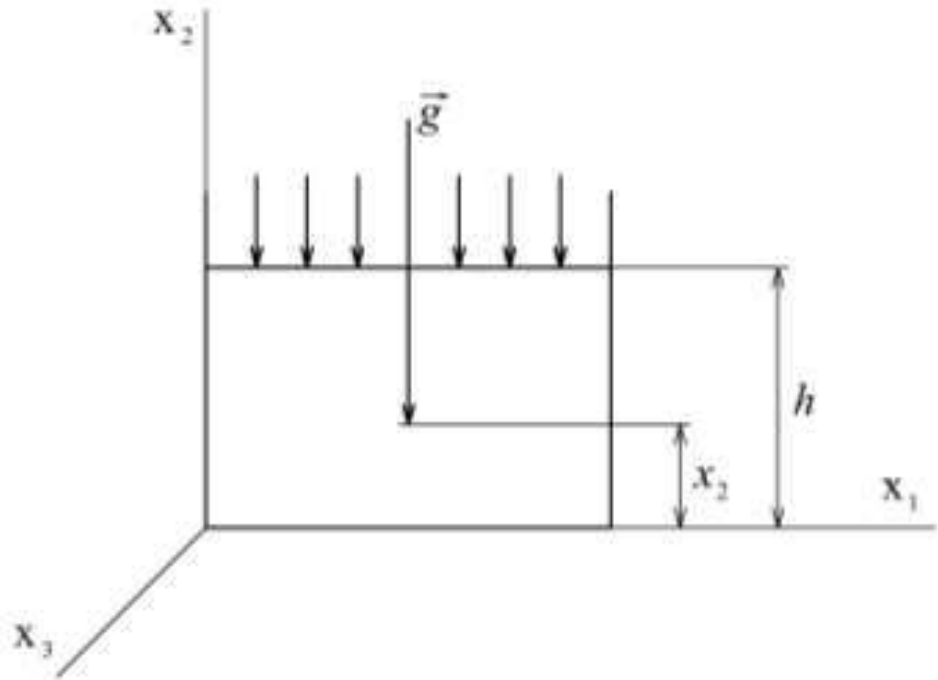
$$\frac{\partial p}{\partial x_1} = 0 \quad \Rightarrow \quad p\left(x_1\right) = \text{konst}$$

$$\frac{\partial p}{\partial x_3} = 0 \quad \Rightarrow \quad p\left(x_3\right) = \text{konst}$$

$$\frac{\partial p}{\partial x_2} = -\rho g$$

$$p = -\rho g x_2 + C$$

$$p\left(h\right) = p_0 \quad \Rightarrow \quad C = p_0 + \rho g h \quad \Rightarrow \quad \boxed{p = p_0 + \rho g\left(h - x_2\right)}$$



# Barometrická formule

$$\rho \vec{a} = \rho \vec{K} - \operatorname{grad} p$$

$$0 = \rho \vec{K} - \operatorname{grad} p$$

$$0 = \rho \vec{g} - \operatorname{grad} p$$

$$\frac{\partial p}{\partial x_1} = 0 \;\Rightarrow\; p(x_1) = \text{konst}$$

$$\frac{\partial p}{\partial x_2} = -\rho g$$

$$\frac{\partial p}{\partial x_2} = -\rho_0 \frac{p}{p_0} g$$

$$\frac{\partial p}{p} = -\frac{\rho_0}{p_0} g \partial x_2$$

izotermická atmosféra $\quad pV = \text{konst}$

$$\frac{pm}{\rho} = \frac{p_0 m}{\rho_0} \;\Rightarrow\; \frac{p}{p_0} = \frac{\rho}{\rho_0}$$

$$\frac{\partial p}{\partial x_3} = 0 \;\Rightarrow\; p(x_3) = \text{konst}$$

$$\ln p = -\frac{\rho_0}{p_0} g x_2 + C$$

$$p = C' \exp\left(-\frac{\rho_0}{p_0} g x_2\right) \quad \Rightarrow \quad C' = p_0$$

$$x_2 = 0 : p = p_0$$

$$\boxed{p = p_0 \exp\left(-\frac{\rho_0}{p_0} g x_2\right)}$$

$$\rho\vec{a} = -\rho\,\mathrm{grad}\,U - \mathrm{grad}\,p$$

$$\vec{K} = -\mathrm{grad}\,U$$

$$dU = -\vec{K}\cdot d\vec{r}$$

$$\vec{a}\cdot d\vec{r} = -dU - \frac{1}{\rho}dp$$

$$\vec{a}\cdot d\vec{r} = d\left(\frac{1}{2}v^2\right)$$

$$d\left(\frac{1}{2}v^2\right) + dU + \frac{1}{\rho}dp = 0$$

$$\frac{1}{2}\rho v^2 + \rho U + p = \mathrm{konst}$$

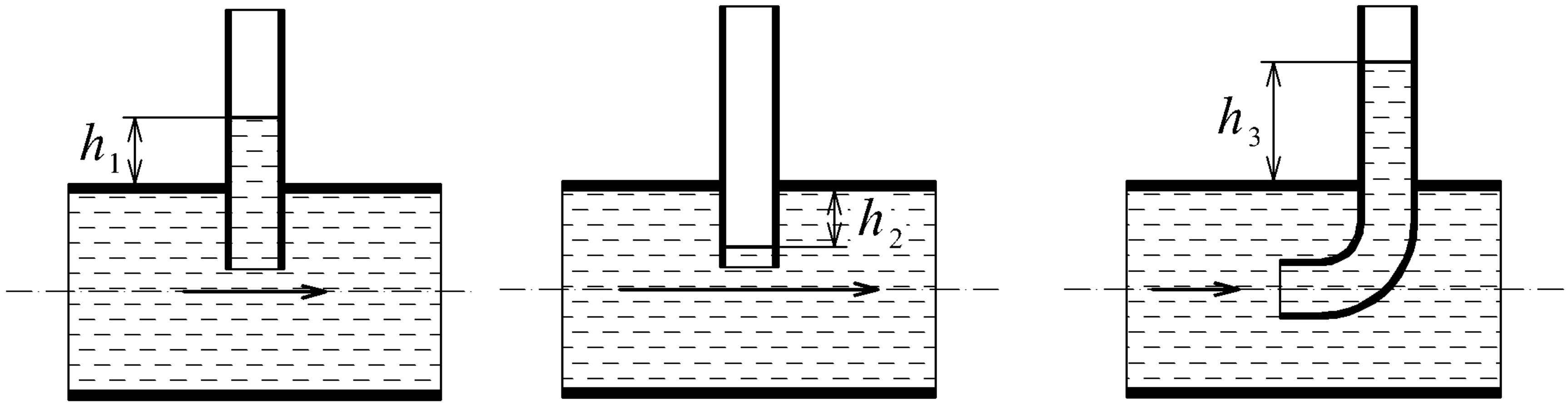
**Bernoulliho rovnice**

(nestlačitelná tekutina)

$$\frac{1}{2}\rho v^2 + \rho g h + p = \mathrm{konst}$$

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$$

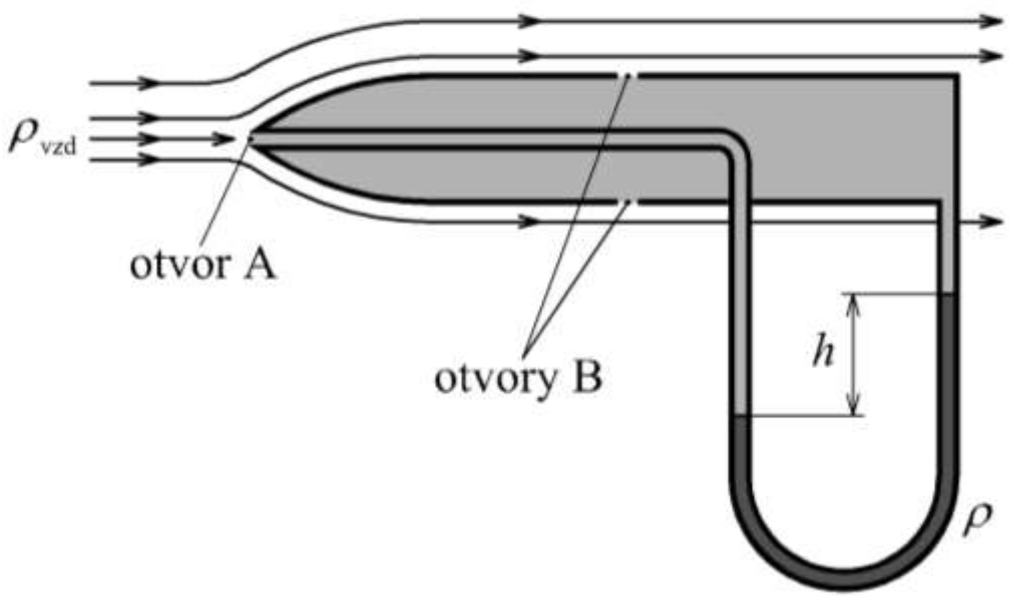
# Pitotova trubice



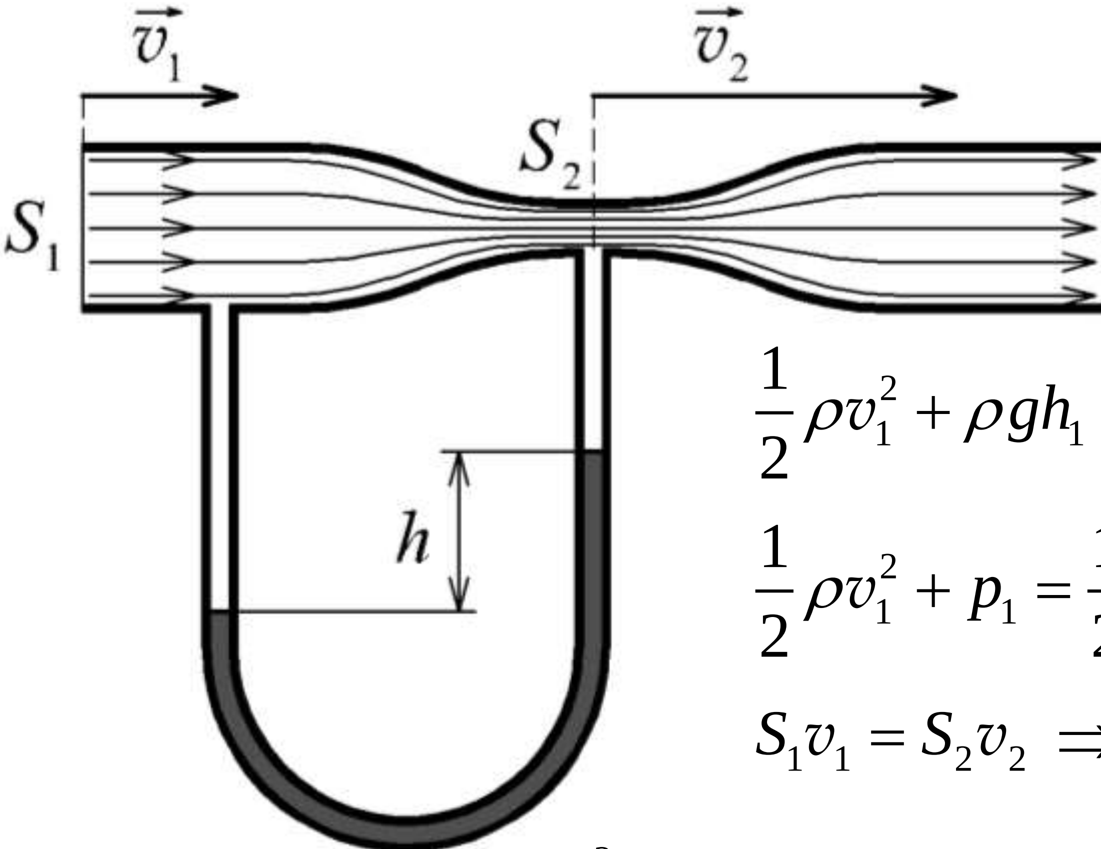

Prandtlova trubice

$$\frac{1}{2}\rho_{\text{vzd}}v^2 + p = p + \rho g h$$

$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{vzd}}}}$$



# Venturiův průtokoměr




$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2 \qquad (h_1 = h_2)$$

$$S_1 v_1 = S_2 v_2 \Rightarrow v_2 = \frac{S_1}{S_2} v_1$$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho \left(\frac{S_1}{S_2}\right)^2 v_1^2 + p_2$$

$$\frac{1}{2}\rho\left(\frac{S_1^2}{S_2^2} - 1\right) v_1^2 = p_1 - p_2 = \Delta p \qquad \Rightarrow v_1 = \sqrt{\frac{2S_2^2 \Delta p}{\rho\left(S_1^2 - S_2^2\right)}} \qquad \Delta p = \rho_n g h$$