Tuhé těleso

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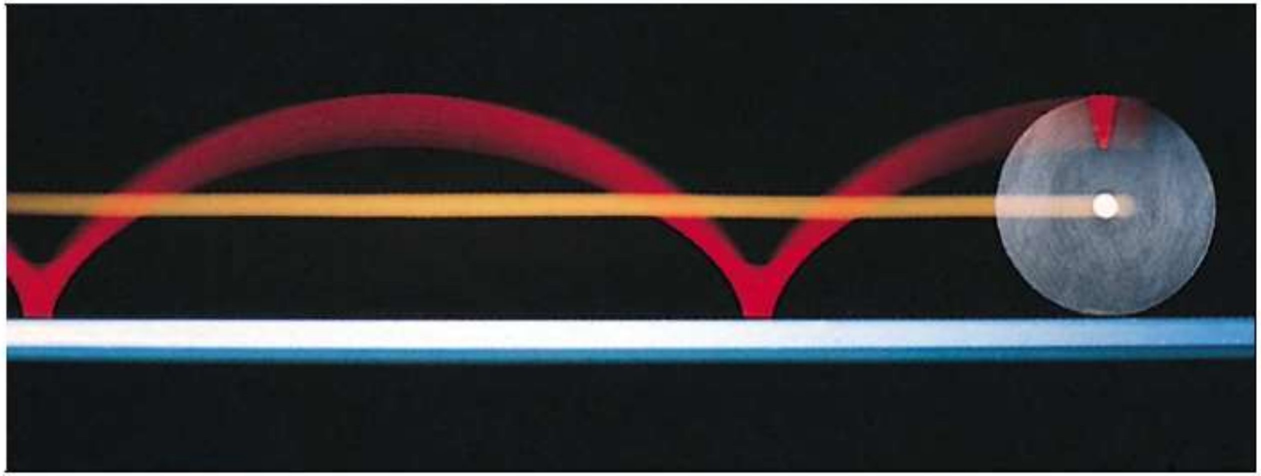
vzdálenost jakýchkoli dvou bodů tělesa je během pohybu konstantní

6 stupňů volnosti

pohyb tuhého tělesa = **translační** pohyb středu hmotnosti +
rotace tělesa kolem okamžité osy procházející středem hmotnosti



$$dm = \rho\, dV$$

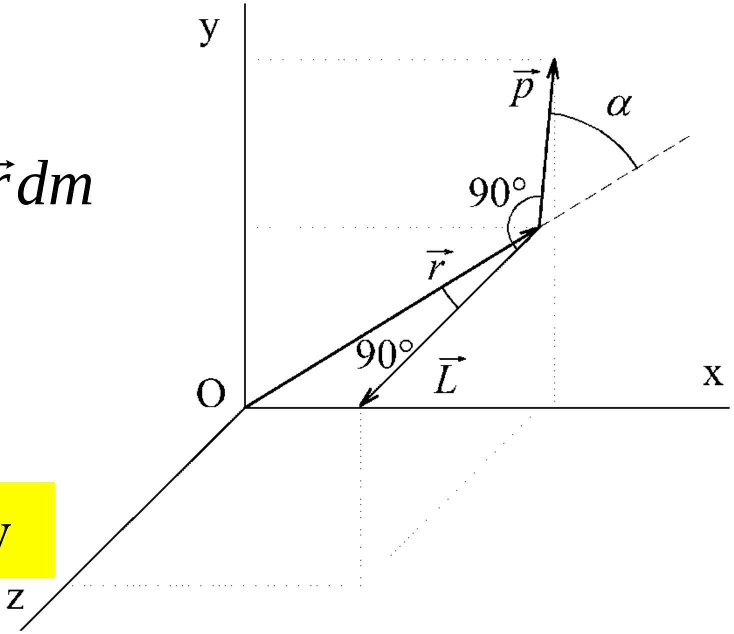
$$m = \int_{(m)} dm = \int_{(V)} \rho\, dV$$

Halliday, Resnik, Walker: Fyzika, Prometheus, 2003

1. věta impulsová

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d\vec{p}_s}{dt} = m\frac{d^2\vec{r}_s}{dt^2} \qquad \vec{r}_s = \frac{1}{m}\int_{(m)} \vec{r}\,dm$$

$$\vec{F} = \frac{d\vec{p}_s}{dt}$$



Rotace tuhého tělesa kolem pevné osy

moment síly $\vec{M} = \vec{r} \times \vec{F}$

$$M = rF\sin\alpha$$

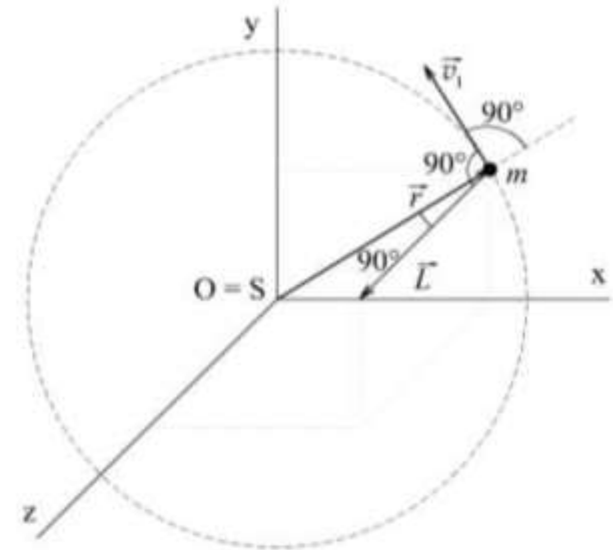
$$\vec{M} = \frac{d\vec{L}}{dt}$$

moment hybnosti $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$$

$$L = \left|\vec{r} \times \vec{p}\right| = \left|\vec{r} \times m\vec{v}\right| = rmv = mr^2\omega$$



$$\vec{L} = \sum_i \vec{L}_i$$

$$\vec{L}_i = m_i r_i^2 \vec{\omega}_i = m_i r_i^2 \vec{\omega}$$

$$\vec{L} = \sum_i \vec{L}_i = \sum_i m_i r_i^2 \vec{\omega}_i = \left(\sum_i m_i r_i^2\right)\vec{\omega} = I\vec{\omega}$$

$\sum_i m_i r_i^2 = I$ moment setrvačnosti soustavy hmotných bodů

$I = \int\limits_{(m)} r^2 dm$ moment setrvačnosti tuhého tělesa

$$\vec{L} = I\vec{\omega}$$

$$\vec{M} = \sum_i \vec{M}_i = \sum_i \frac{d\vec{L}_i}{dt} = \frac{d}{dt}\left(\sum_i \vec{L}_i\right) = \frac{d\vec{L}}{dt}$$

2. věta impulsová

$$\vec{M} = \sum_i \vec{M}_i = \frac{d\vec{L}}{dt}$$

$$\vec{L} = I\vec{\omega}$$

$$\vec{M} = \frac{d\vec{L}}{dt} = \frac{d}{dt}\left(I\vec{\omega}\right) = I\,\frac{d\vec{\omega}}{dt}$$

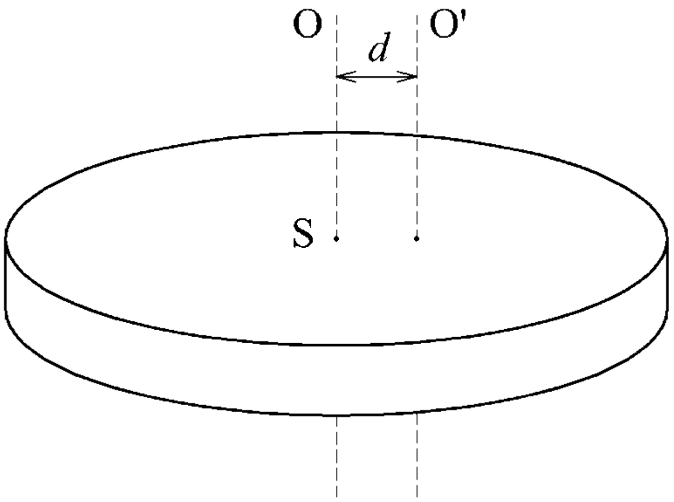
$$\vec{M} = I\vec{\varepsilon}$$

$$I = \int_{(m)} r^2 dm$$

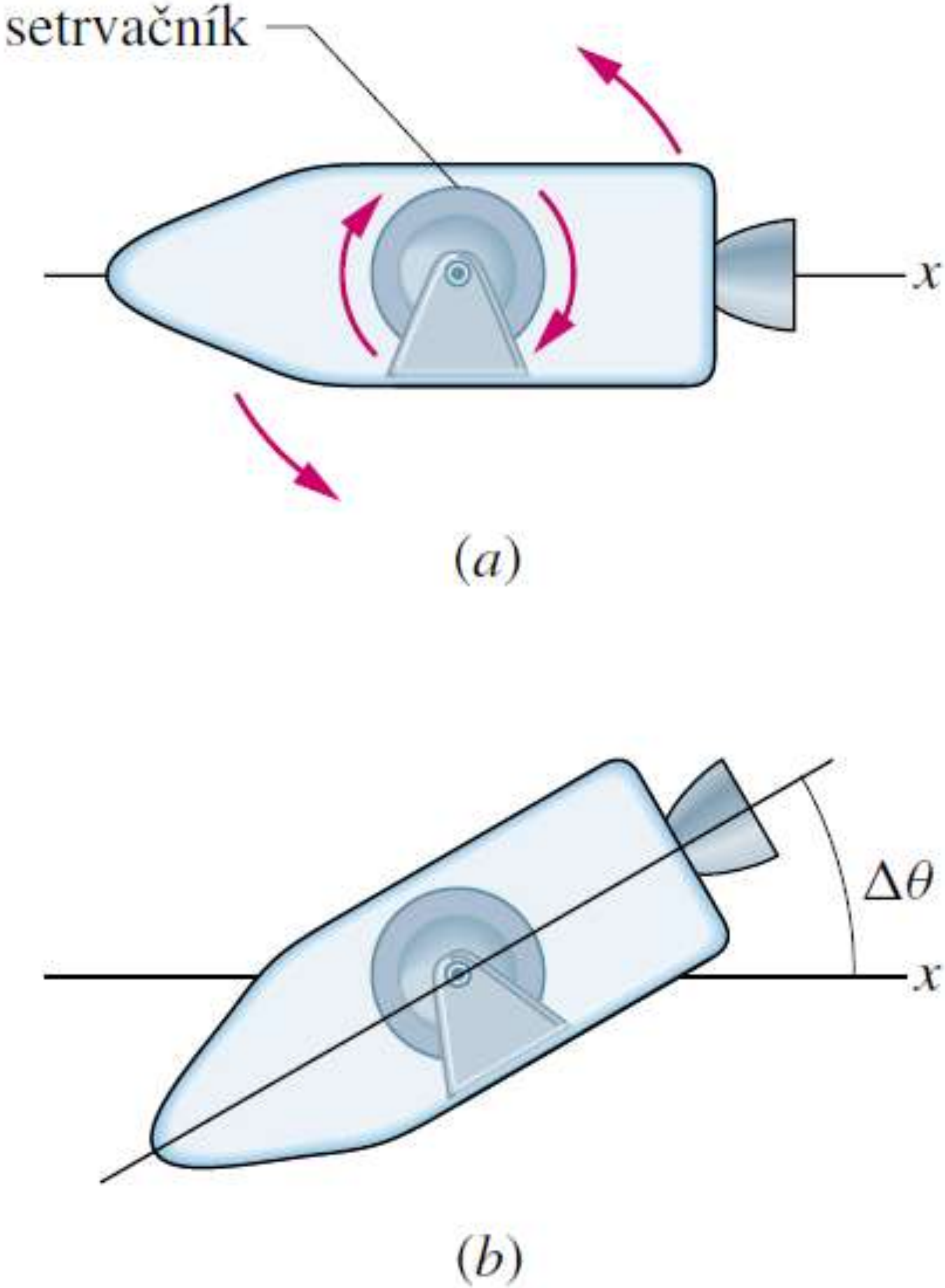
$$\vec{M} = \sum_i \vec{M}_i = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \overrightarrow{\mathrm{konst}}$$

zákon zachování momentu hybnosti

Steinerova věta

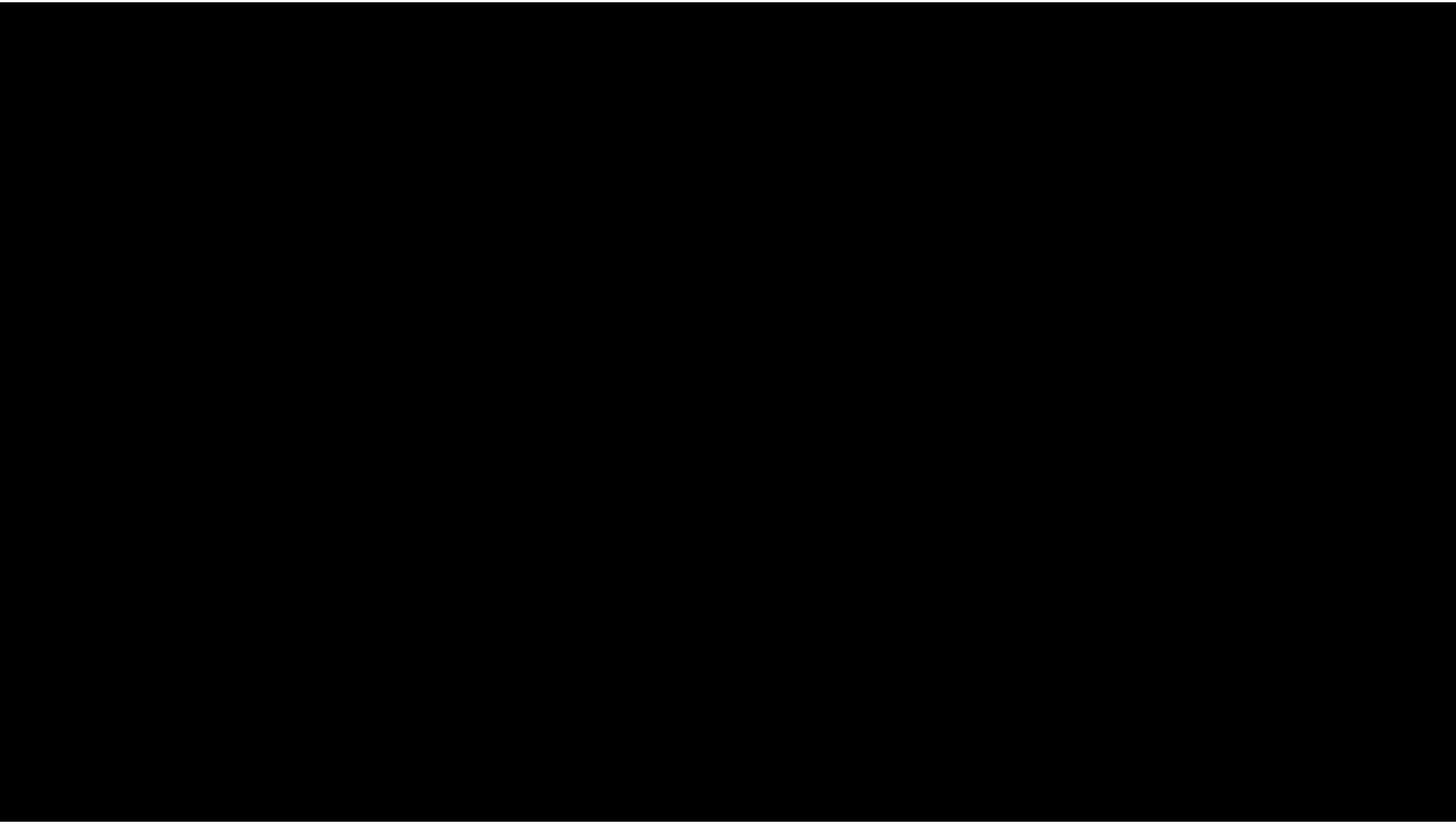


$$I = I_0 + md^2$$

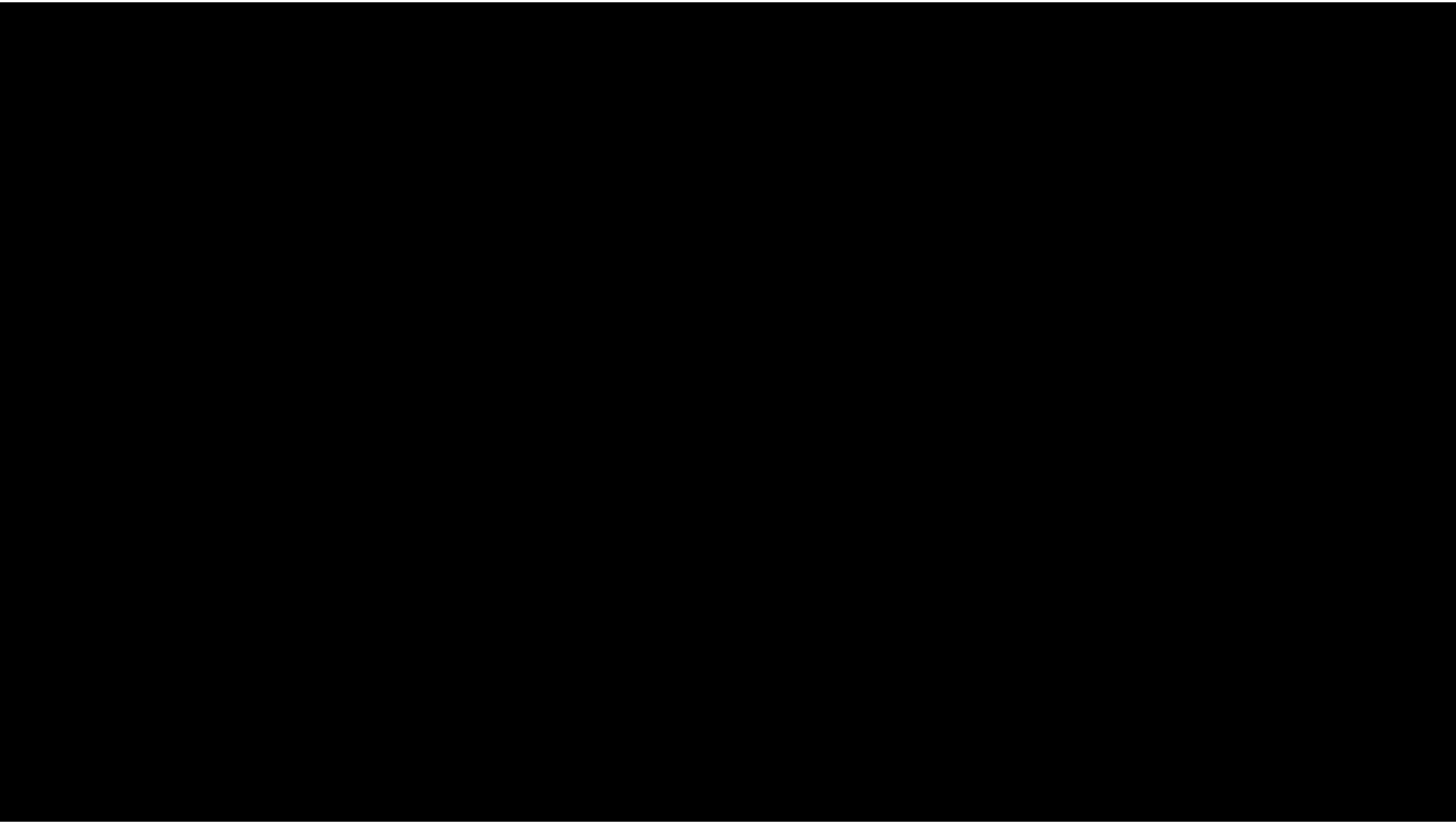


Halliday, Resnik, Walker:
Fyzika, Prometheus, 2003

Moment hybnosti



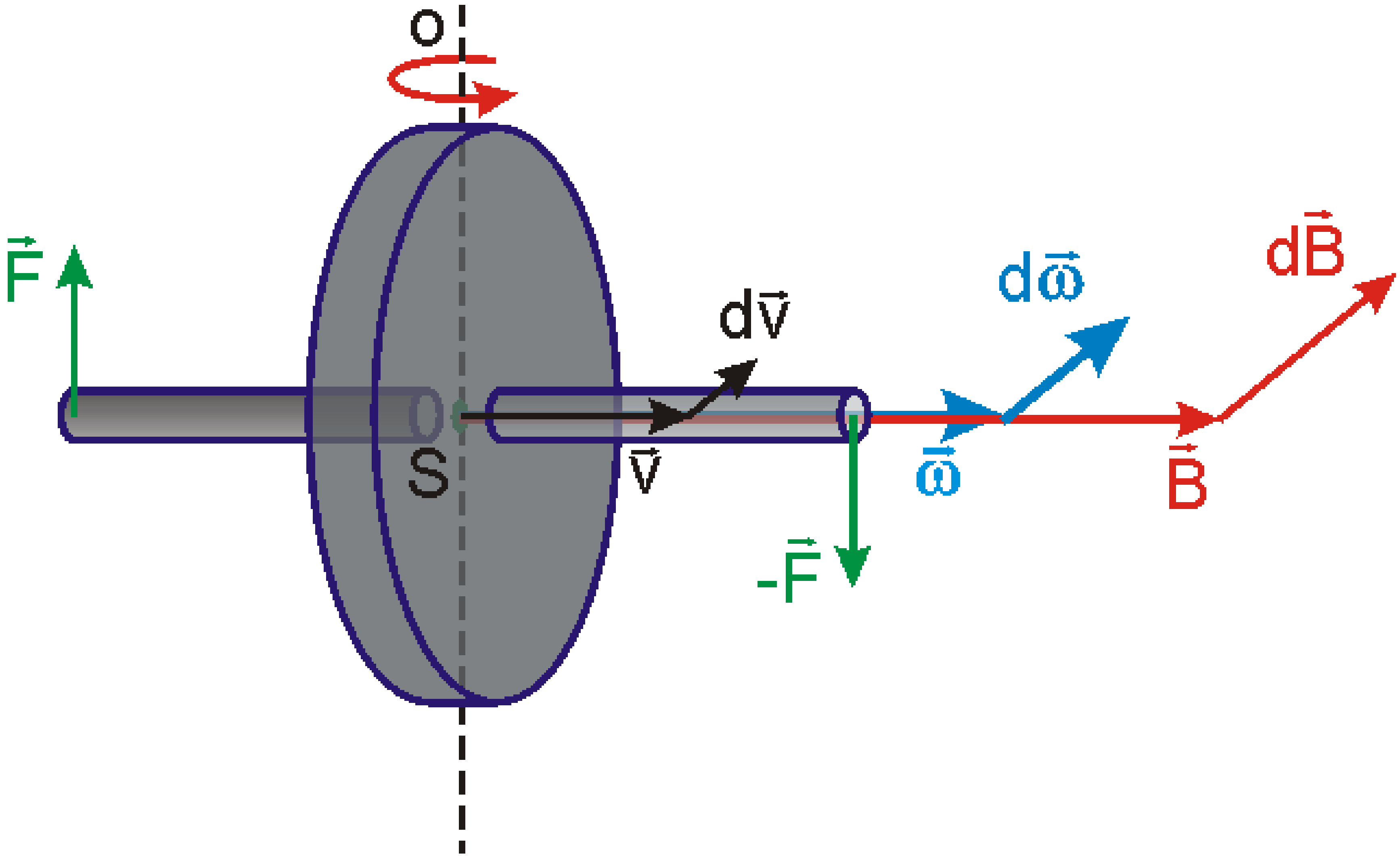
https://www.fd.cvut.cz/personal/malazuza/prednasky.html



http://fyzweb.cz/materialy/videopokusy/POKUSY/SETRVACNIK/INDEX.HTM

https://www.youtube.com/watch?v=xQb-N486mA4

https://www.matfyz.cz/clanky/fyzikalni-pokus-gyroskop



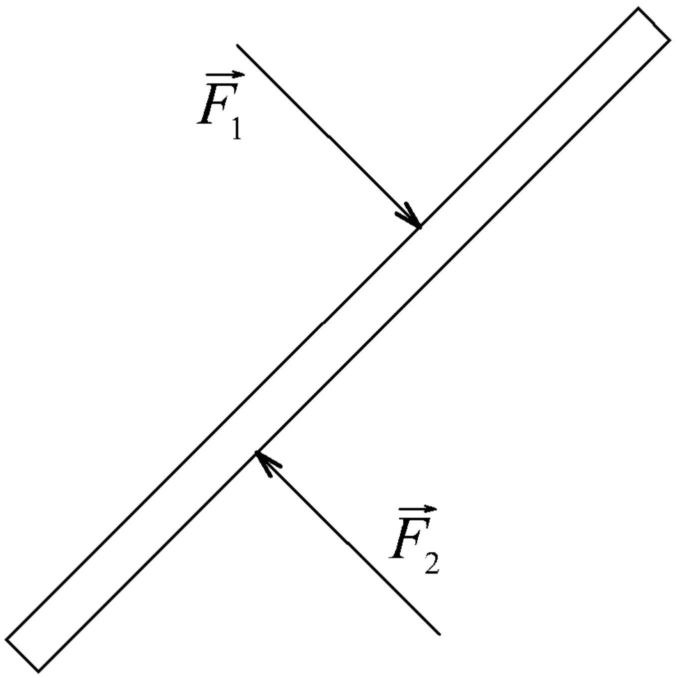
http://physics.mff.cuni.cz/kfpp/skripta/kurz_fyziky_pro_DS/display.php/mechanika/6_6

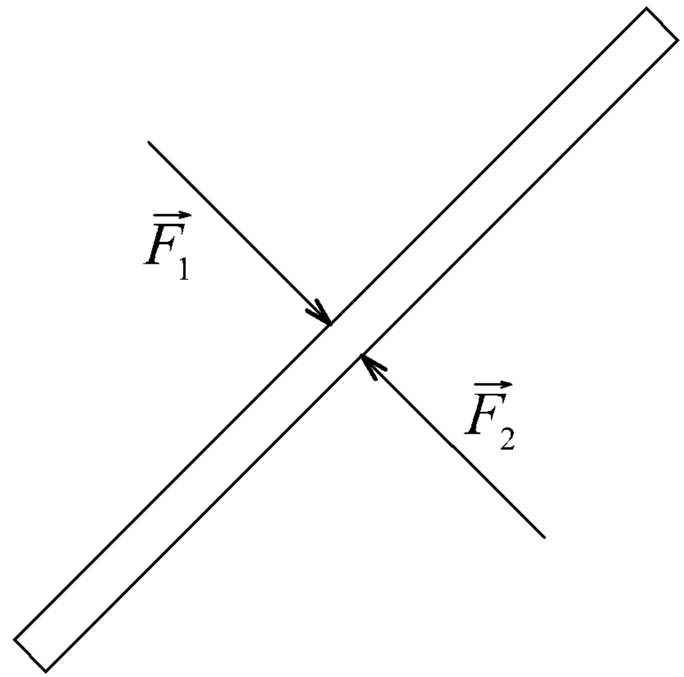
Statická rovnováha tuhého tělesa

$$\sum_i \vec{F}_i = 0 \qquad \sum_i \vec{M}_i = \sum \left(\vec{r}_i \times \vec{F}_i \right) = 0$$

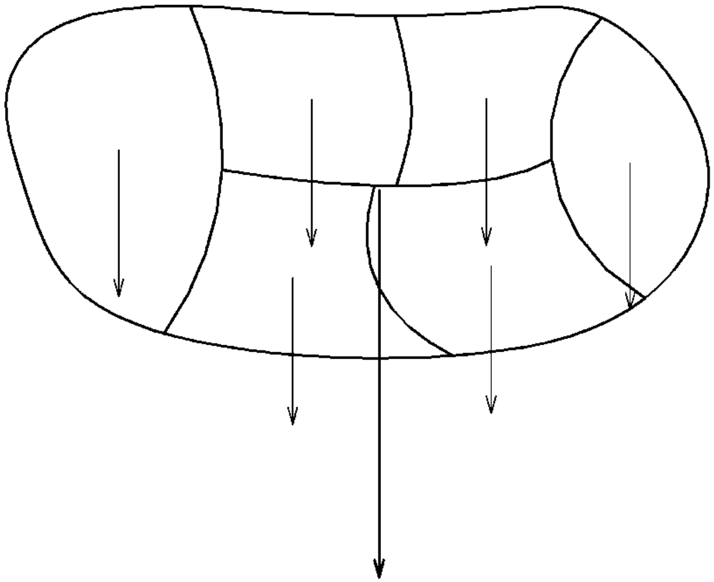
Silová dvojice

$$M = Fd$$





Těžiště – působiště tíhové síly



$$\vec{F}_g = \sum_i m_i \vec{g}$$

$$\vec{r}_T \times \vec{F}_g = \sum_i \left(\vec{r}_i \times m_i \vec{g} \right)$$

$$\vec{r}_T m \times \vec{g} = \sum_i \left(\vec{r}_i m_i \times \vec{g} \right)$$

$$\vec{r}_T = \frac{1}{m} \int_{(m)} \vec{r}\, dm$$

$$\vec{r}_T = \frac{1}{m} \sum_i \vec{r}_i m_i$$

poloha těžiště – (pro tíhové pole) totožná se středem hmotnosti soustavy

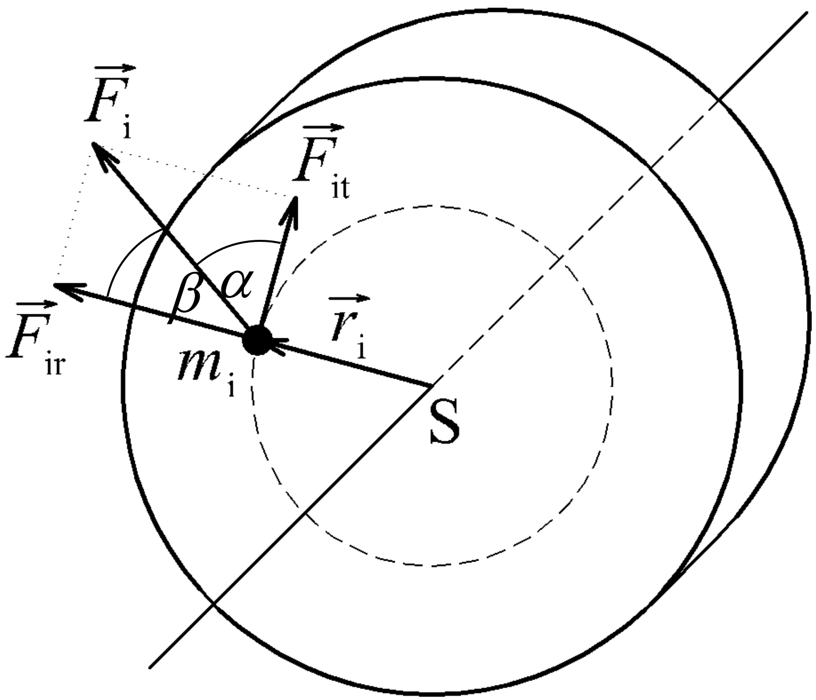
$$\vec{r}_T = \vec{r}_s$$

Kinetická energie tělesa rotujícího kolem pevné osy

$$dA_i = \vec{F}_i \cdot d\vec{r}_i = F_i \cos\alpha \, dr_i = F_i \cos\alpha \, ds_i = F_i \cos\alpha \, r_i \, d\varphi = F_i \sin\beta \, r_i \, d\varphi = M_i \, d\varphi$$

$$ds_i = r_i \, d\varphi$$

$$dA = \sum_i dA_i = \sum_i M_i \, d\varphi = M \, d\varphi$$



$$P = \frac{dA}{dt} = M\omega$$

$$P = \vec{M} \cdot \vec{\omega}$$

$$W_{kR} = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} I \omega^2$$

$$W_k = W_{kT} + W_{kR}$$

$$W_k = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v_s^2 + \frac{1}{2} I \omega^2$$