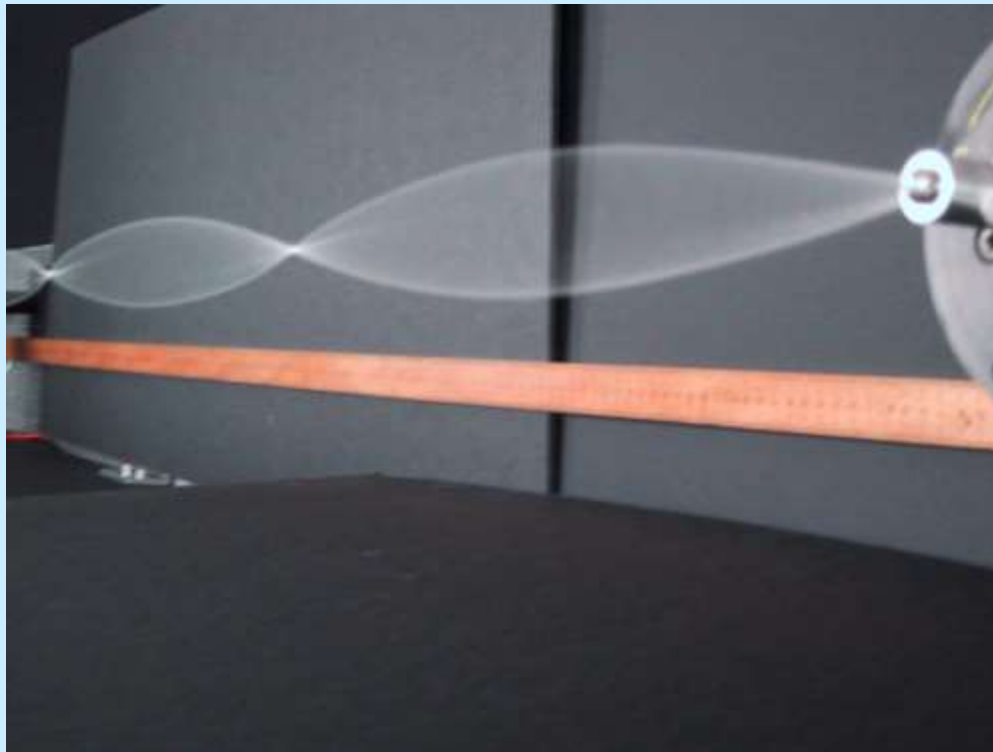


Reálné tekutiny

Vlnění



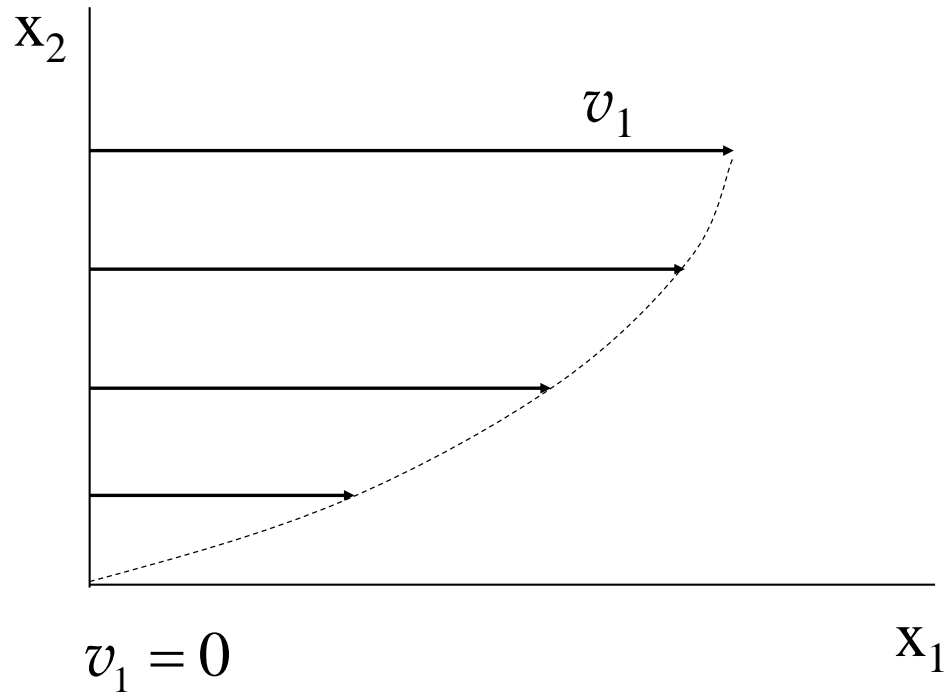
<http://saaphysics.com/PhysicsText/Ch1Sect2.htm>

Viskozita

$$\sigma_{v_{21}} = \eta \frac{\partial v_1}{\partial x_2}$$

$$\sigma_{vik} = \eta \frac{\partial v_k}{\partial x_i}$$

Newtonův zákon

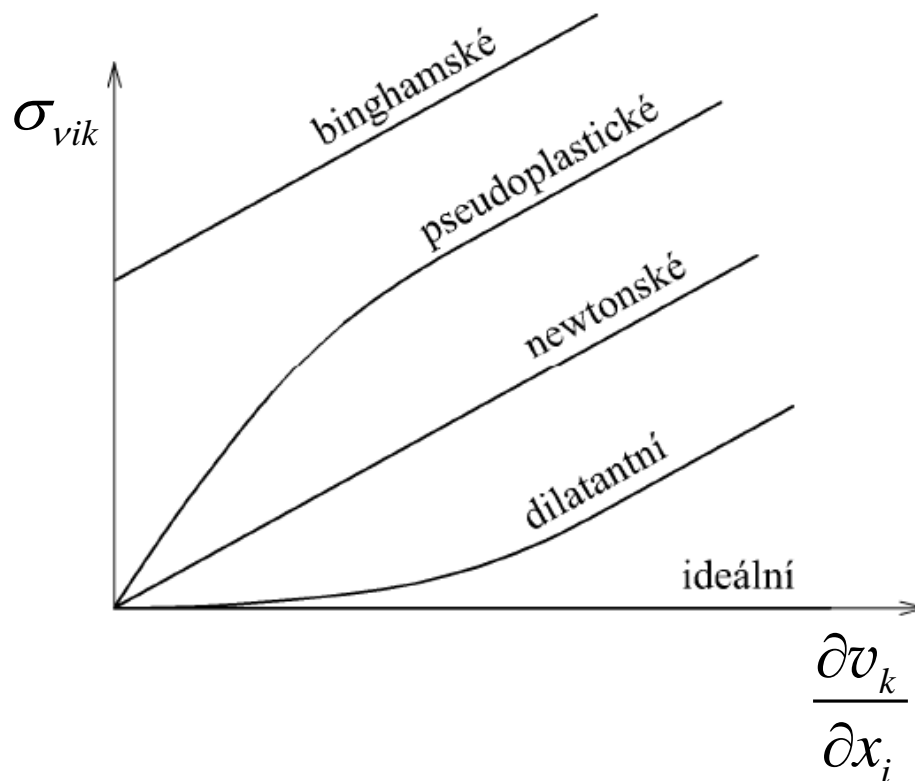


η koeficient dynamické viskozity $[\eta] = \text{Pa} \cdot \text{s}$

$\nu = \frac{\eta}{\rho}$ kinematická viskozita $[\nu] = \text{m}^2 \cdot \text{s}^{-1}$

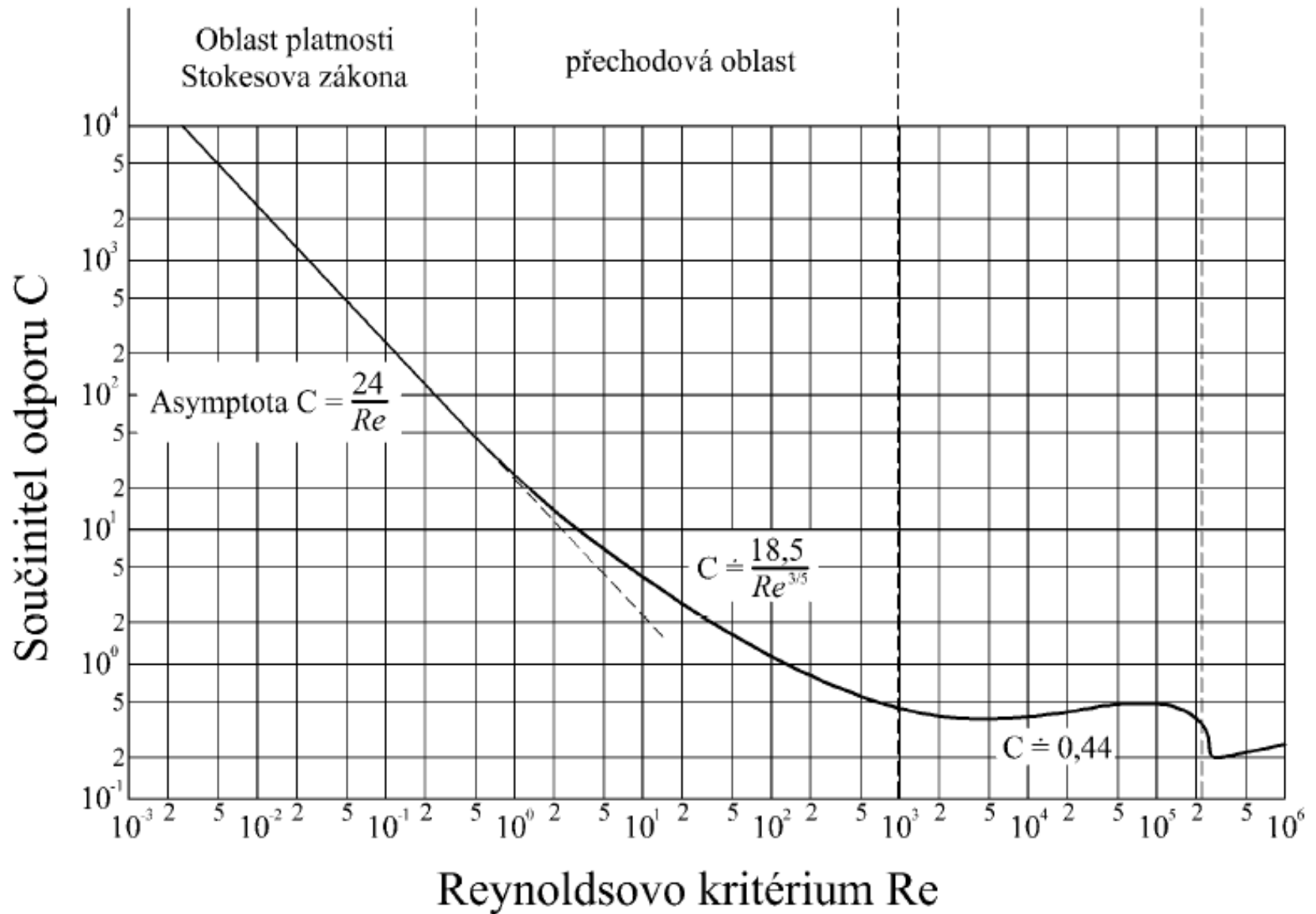
Podle průběhu reogramů se neneutonské tekutiny dělí na:

1. **plastické** (binghamské) – za běžných podmínek nevykazují tekutost, vyžadují překročení určité meze tečného napětí, aby došlo k tečení (kašovitá suspenze, kaly, vápno),
- 2 **pseudoplastické** – při malých rychlostních spádech mají anomální chování a při větších se chovají jako tekutiny newtonské (zředěná suspenze),
3. **dilatantní** – viskozita se se stoupajícím tečným napětím zvětšuje, látky snadno sedimentují (mokrý písek),
4. **tixotropní** – se zvyšujícím se tečným napětím viskozita klesá (jogurt, tixotropní barvy – pod tlakem štětce se stávají snadno roztíratelné),
5. **reopexní** – vlivem zvyšujícího se tečného napětí viskozita stoupá (suspenze, sádry, šlehaný bílek).



$$F = 6\pi\eta r v \text{ Stokesův vzorec} \quad \text{Re} = \frac{vd}{\nu}$$

$$F = \frac{1}{2} \rho C S_p v^2$$

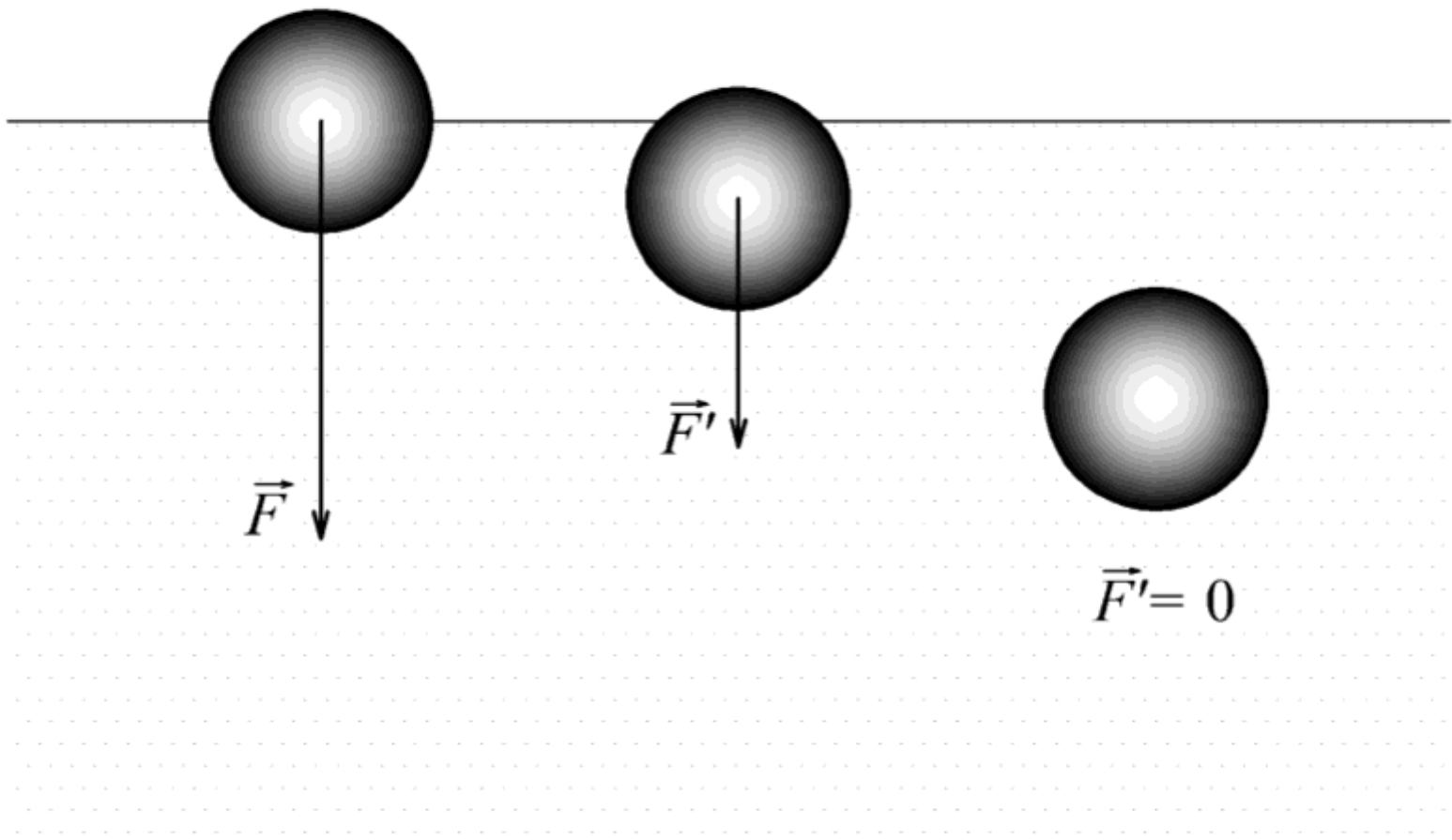


Navierova – Stokesova rovnice pro nestlačitelnou kapalinu

$$\frac{\partial v_i}{\partial t} + \sum_{k=1}^3 \frac{\partial v_i}{\partial x_k} v_k = g_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 v_i$$

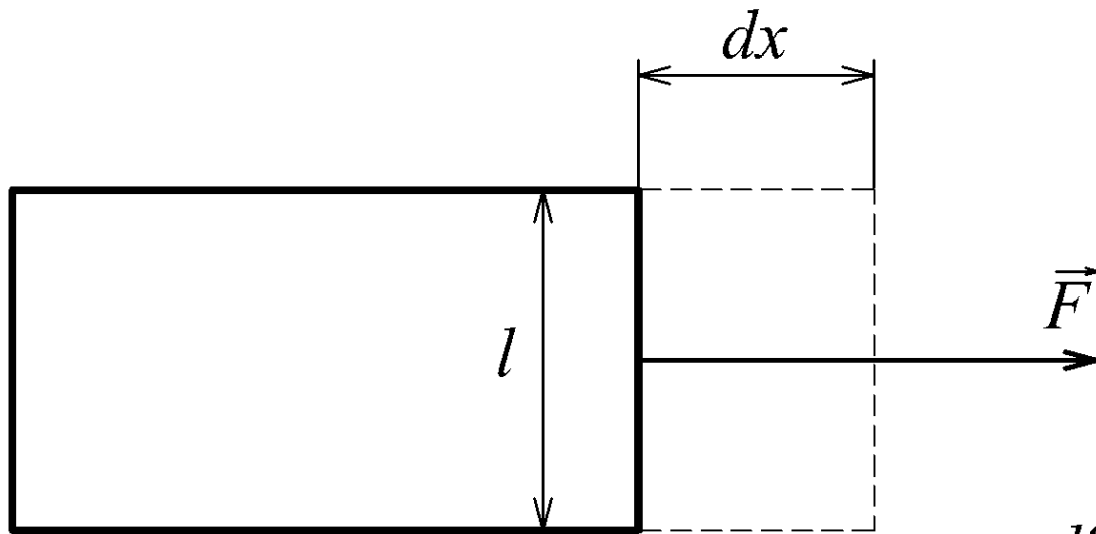
$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

Povrchové jevy v kapalinách



Povrchové napětí

(práce dA interakčních sil mezi molekulami na vytvoření plochy dS)



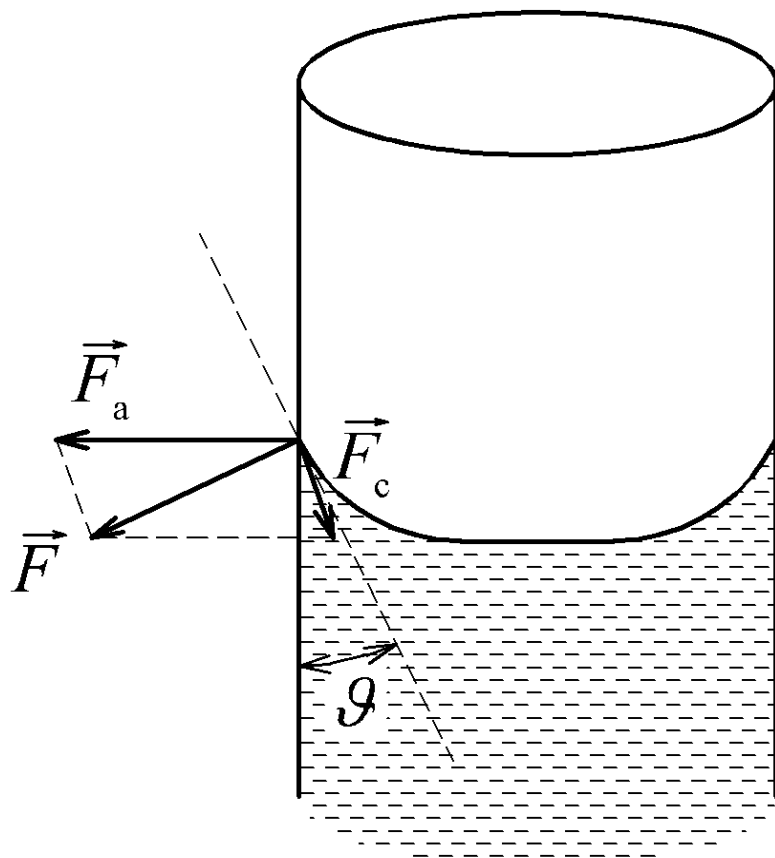
$$dA = \vec{F} \cdot d\vec{r}$$

$$dA = F dx$$

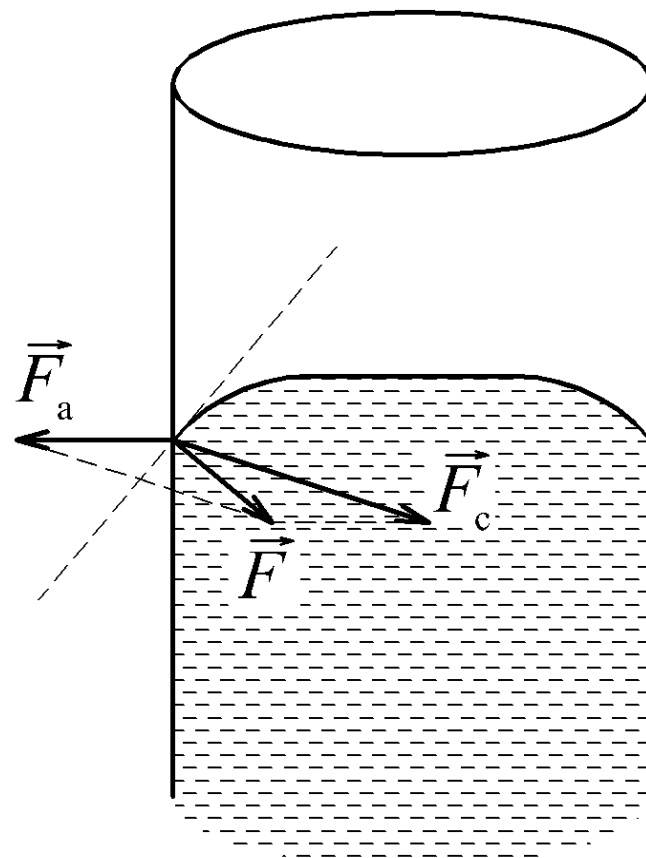
$$\sigma = \frac{\partial A}{\partial S}$$

$$\sigma = \frac{F}{l}$$

$$dS = l dx$$

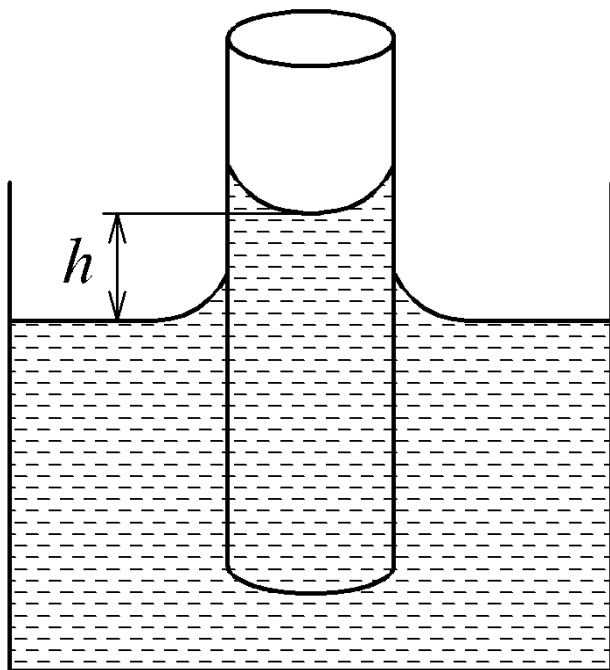


smáčí stěnu $\sigma_{13} > \sigma_{12}$

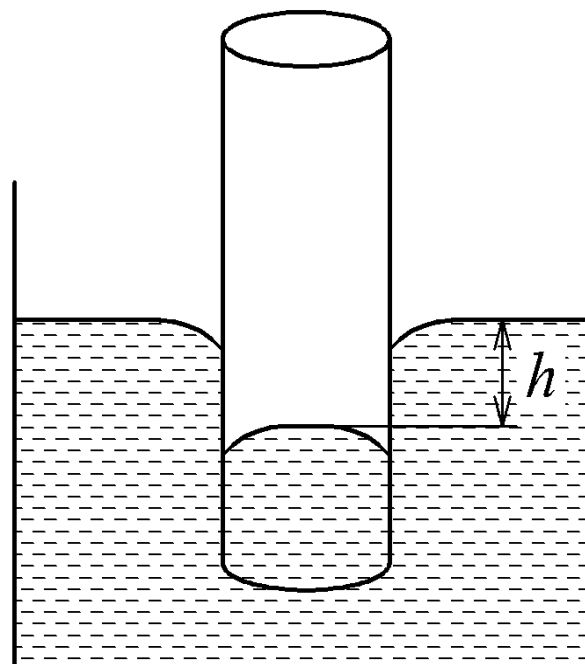


nesmáčí stěnu $\sigma_{13} < \sigma_{12}$

1 – materiál nádoby, 2 – kapalina, 3 – prostředí nad kapalinou



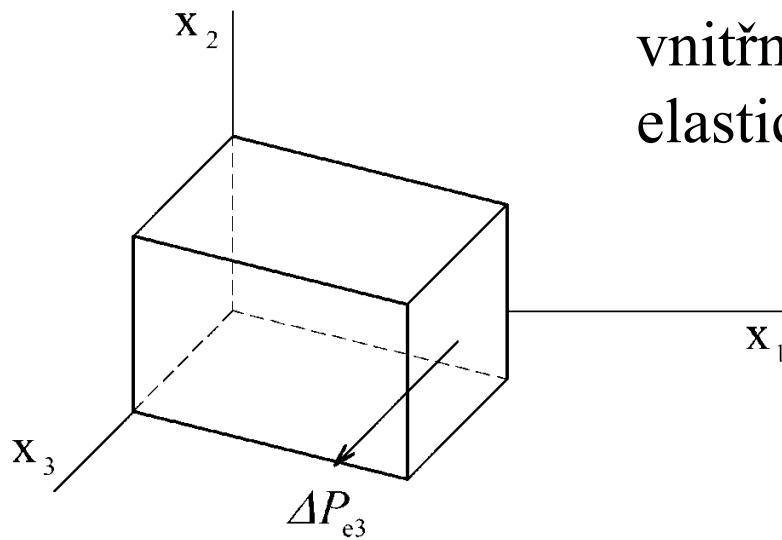
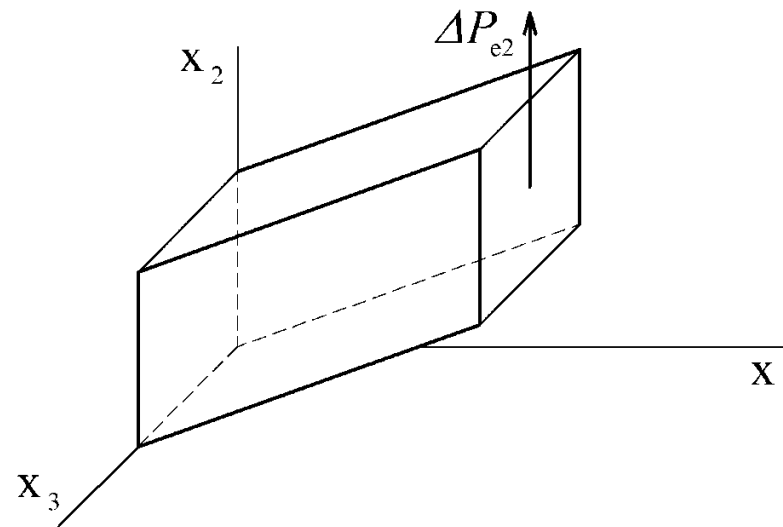
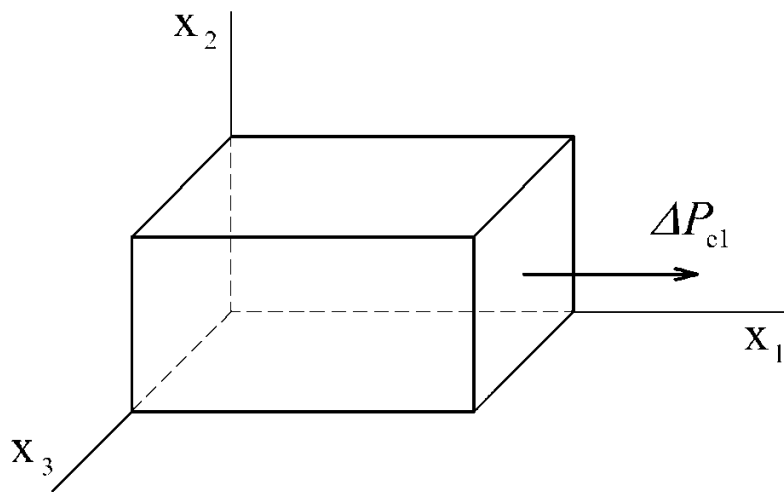
kapilární elevace



kapilární deprese

elastická vlna

vlnění podélné, příčné



vnitřní plošná síla působící
elastické napětí $\Delta \vec{P}_e$

$$\Delta m \vec{a} = \Delta \vec{P}_e \quad \rho \vec{a} = \vec{f}_e \quad \rho \frac{\partial^2 u_i}{\partial t^2} = f_{ei}$$

$$f_i = \sum_{k=1}^3 \frac{\partial \sigma_{ki}}{\partial x_k} = \frac{\partial \sigma_{1i}}{\partial x_1}$$

$$\sigma_{ii} = E \frac{\partial u_i}{\partial x_i}$$

$$\sigma_{11} = E \frac{\partial u_1}{\partial x_1}$$

$$\sigma_{ki} = G \frac{\partial u_i}{\partial x_k}$$

$$\sigma_{1i} = G \frac{\partial u_i}{\partial x_1}$$

$$\Delta m \vec{a} = \Delta \vec{P}_e \quad \rho \vec{a} = \vec{f}_e \quad \rho \frac{\partial^2 u_i}{\partial t^2} = f_{ei} \quad f_{ei} = \sum_{k=1}^3 \frac{\partial \sigma_{ki}}{\partial x_k} = \frac{\partial \sigma_{1i}}{\partial x_1}$$

$$\frac{\partial^2 u_1}{\partial x_1^2} = \frac{\rho}{E} \frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial^2 u_2}{\partial x_1^2} = \frac{\rho}{G} \frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial^2 u_3}{\partial x_1^2} = \frac{\rho}{G} \frac{\partial^2 u_3}{\partial t^2}$$

vlnová rovnice

Vlnová rovnice

$$\frac{\partial^2 u_1}{\partial x_1^2} = \frac{\rho}{E} \frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial^2 u_2}{\partial x_1^2} = \frac{\rho}{G} \frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial^2 u_3}{\partial x_1^2} = \frac{\rho}{G} \frac{\partial^2 u_3}{\partial t^2}$$

$$c_1 = \sqrt{\frac{E}{\rho}}$$

$$c_2 \equiv c_3 = \sqrt{\frac{G}{\rho}}$$

rovinná harmonická vlna

$$u_i(x_1, t) = u_0 \sin \omega \left(t - \frac{x_1}{c_i} \right)$$

$$u_i(x_1, t) = u_0 \sin 2\pi \left(\frac{t}{T} - \frac{x_1}{\lambda_i} \right)$$

$$u_i(x_1, t) = u_0 \sin(\omega t - k_i x_1)$$

$$u(\vec{r}, t) = u_0 \sin(\omega t - \vec{k} \cdot \vec{r})$$

$$T = \frac{1}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\lambda = cT = \frac{c}{f}$$

$$k = \frac{2\pi}{\lambda}$$

$$\vec{k} = (k_1; k_2; k_3)$$

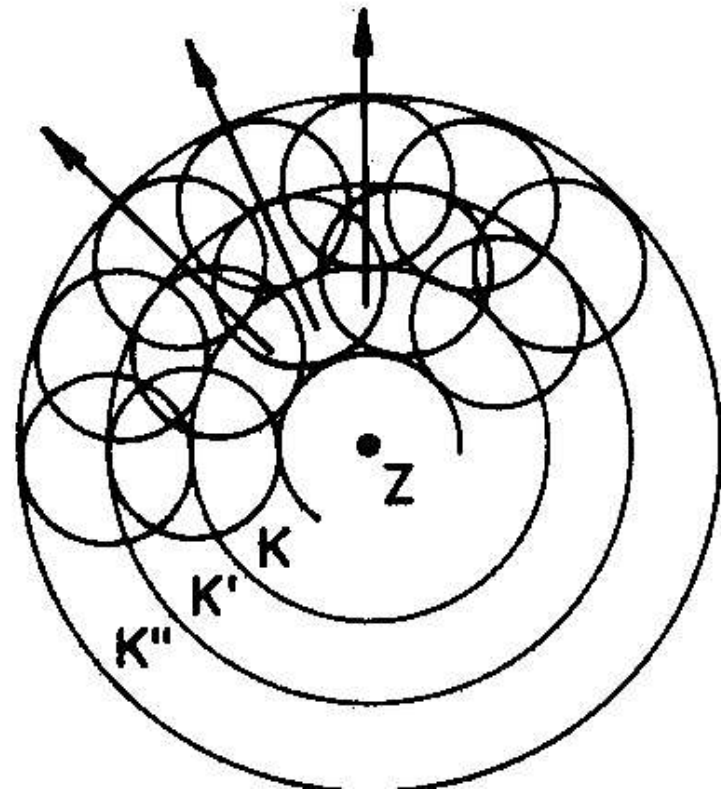
$$\frac{\omega}{c} = k$$

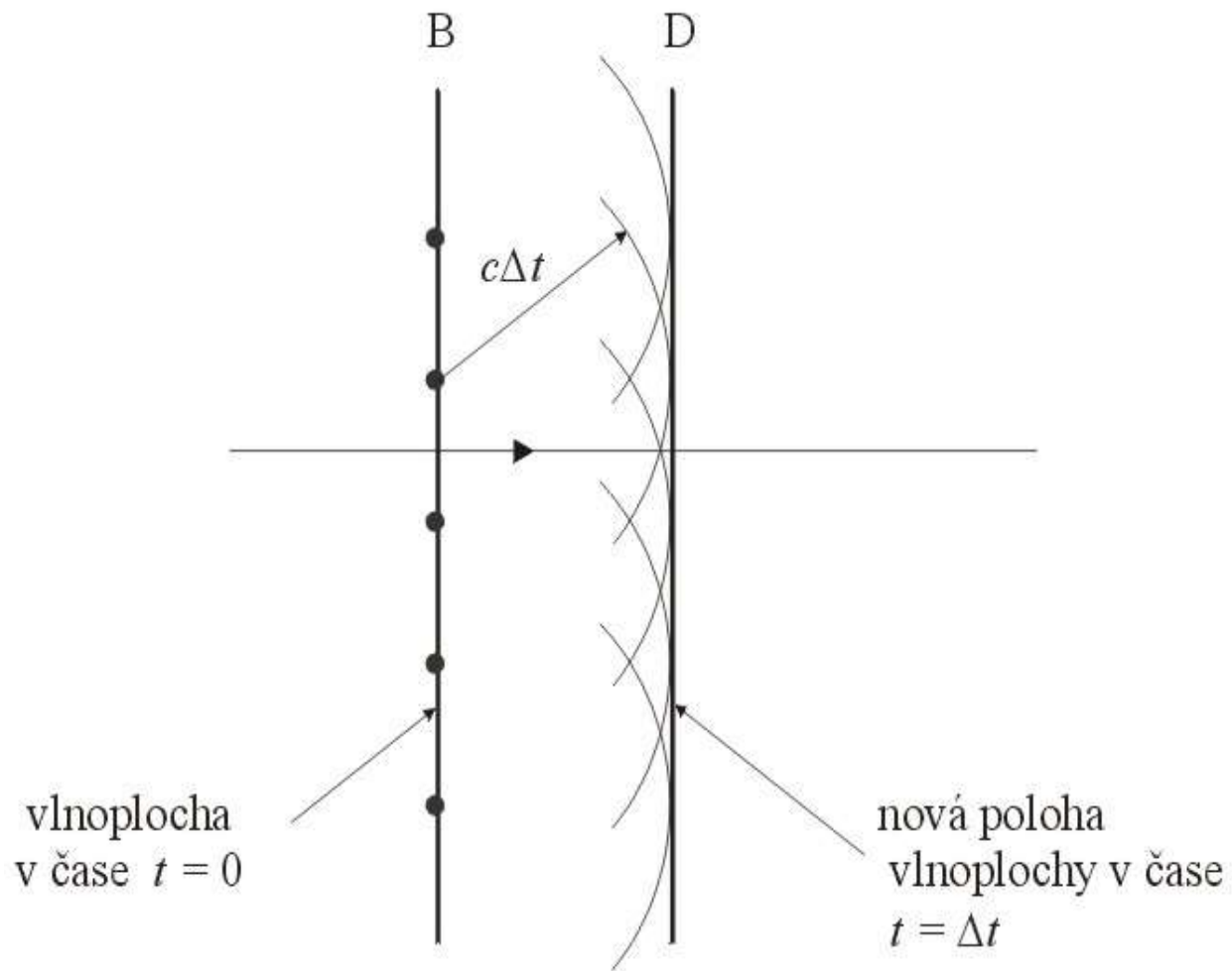
homogenní izotropní prostředí

vlnoplocha

Huygensův princip

elementární zdroj vlnění





Interference vlnění

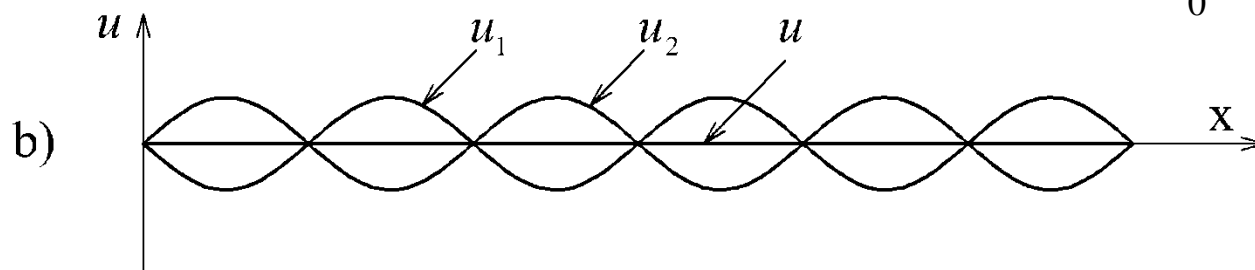
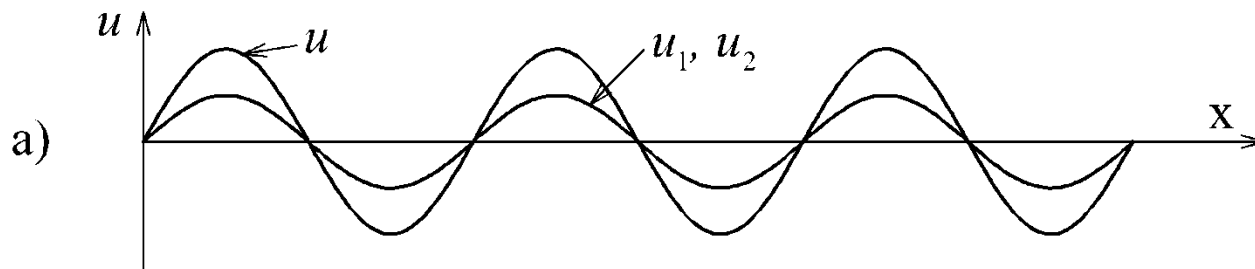
princip superpozice

$$u_1 = u_0 \sin(\omega t - kx_1)$$

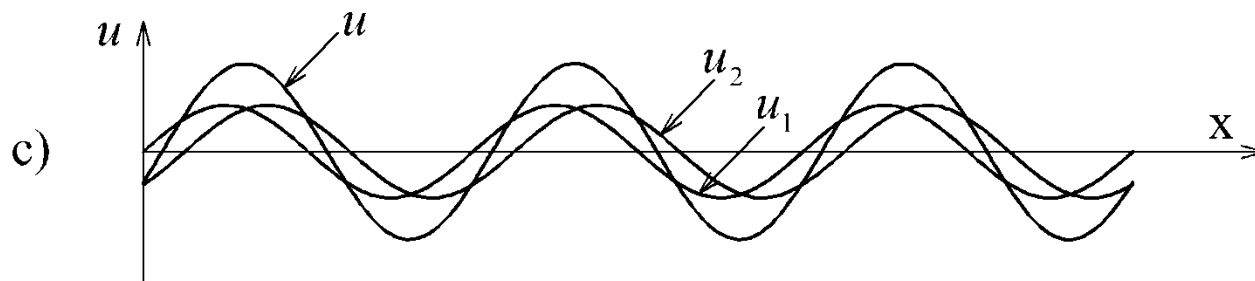
$$u_2 = u_0 \sin(\omega t - kx_1 - \Phi)$$

$$\begin{aligned} u &= u_1 + u_2 = u_0 \sin(\omega t - kx_1) + u_0 \sin(\omega t - kx_1 - \Phi) = \\ &= u_0 \left[\sin(\omega t - kx_1) + \sin(\omega t - kx_1 - \Phi) \right] = \\ &= 2u_0 \cos \frac{\Phi}{2} \sin \left(\omega t - kx_1 - \frac{\Phi}{2} \right) \end{aligned}$$

$$u_1 = u_0 \sin(\omega t - kx_1) \quad u_2 = u_0 \sin(\omega t - kx_1 - \Phi)$$

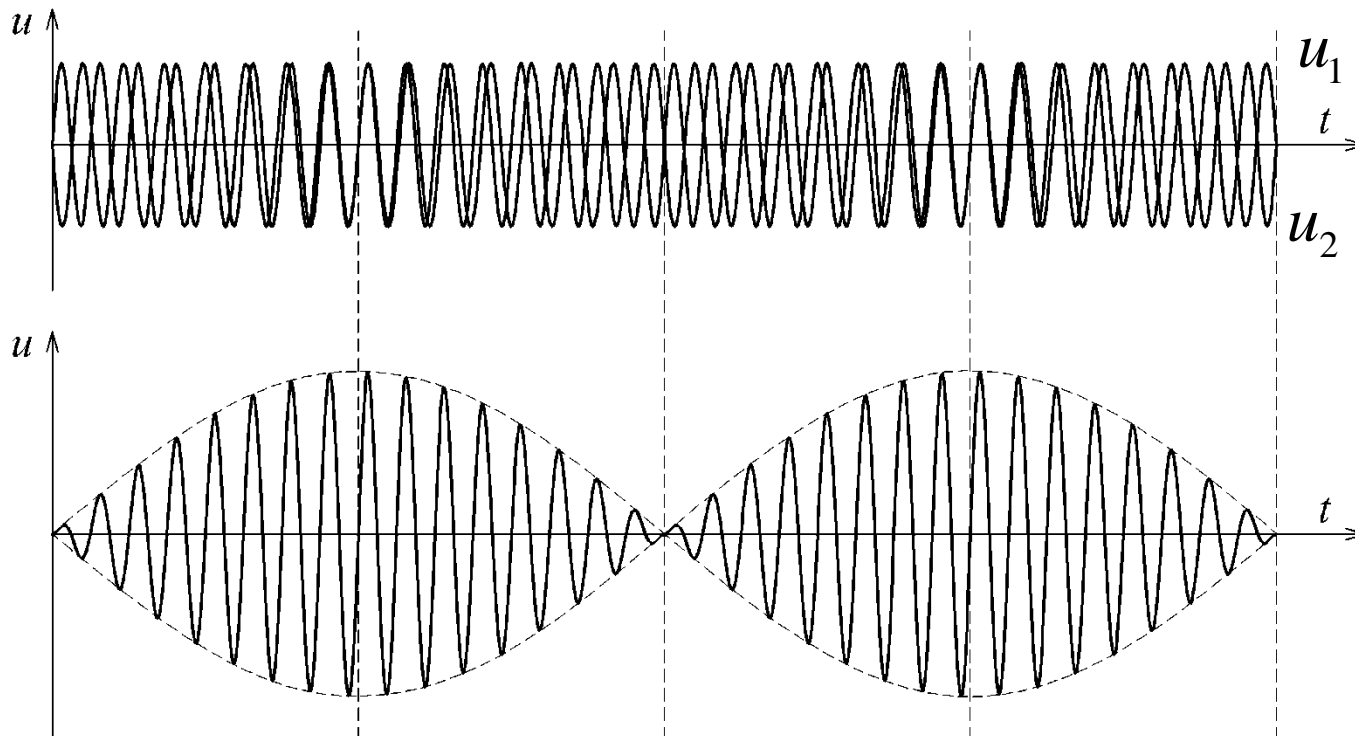


$$u = 2u_0 \cos \frac{\Phi}{2} \sin \left(\omega t - kx_1 - \frac{\Phi}{2} \right)$$



Interference vlnění s blízkými frekvencemi

rázy



$$u_1 = u_0 \sin(\omega_1 t - k_1 x_1)$$

$$u_2 = u_0 \sin(\omega_2 t - k_2 x_1)$$

$$\omega_2 = \omega_1 + \Delta\omega$$

$$k_2 = k_1 + \Delta k$$

$$v_f = \frac{\omega}{k}$$

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

$$u_1 + u_2 = 2u_0 \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_1\right) \sin\left(\left(\omega_1 + \frac{\Delta\omega}{2}\right)t - \left(k_1 + \frac{\Delta k}{2}\right)x_1\right)$$

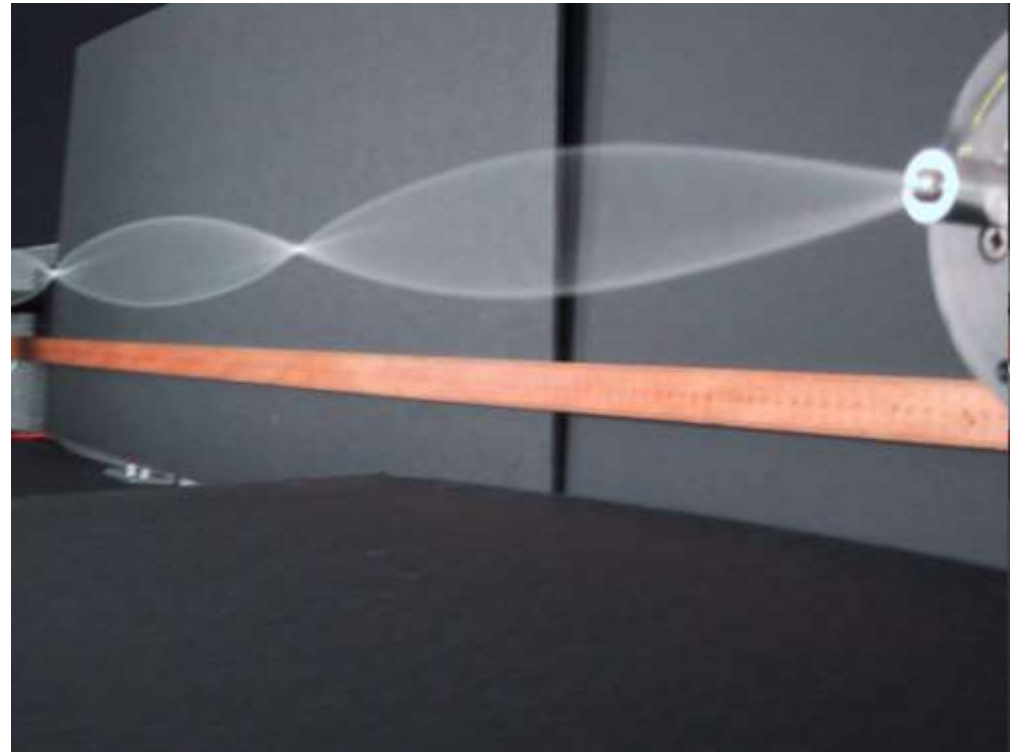
Stojaté vlnění

$$u_1 = u_0 \sin(\omega t - kx_1)$$

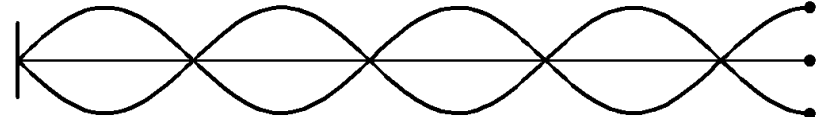
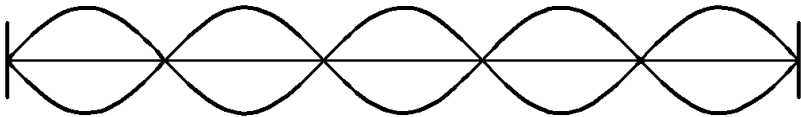
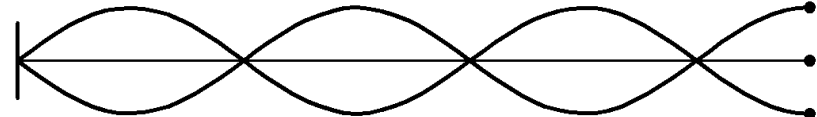
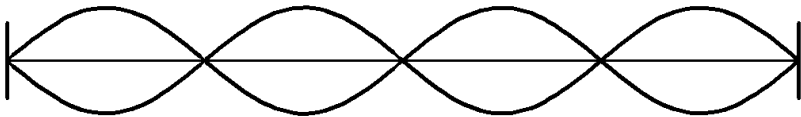
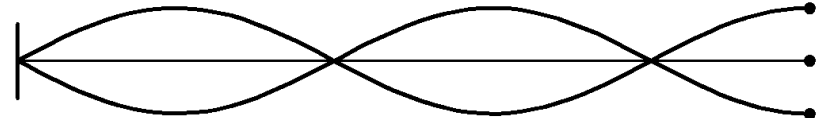
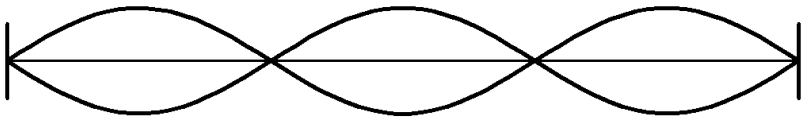
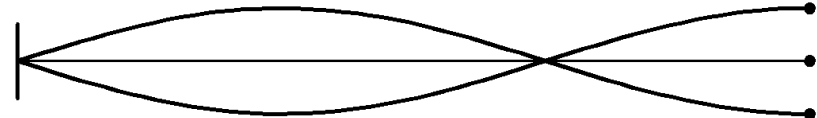
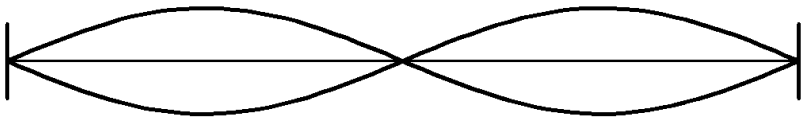
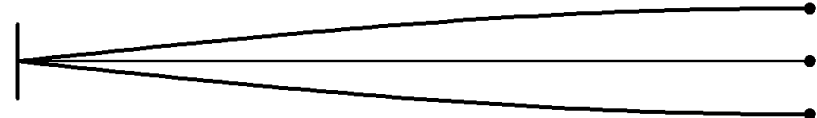
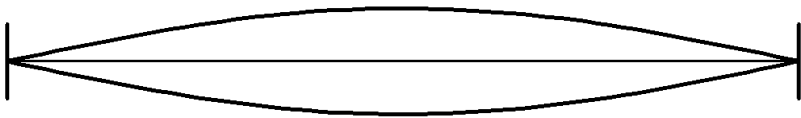
$$u_2 = u_0 \sin(\omega t + kx_1)$$

$$u = u_1 + u_2 = 2u_0 \cos kx_1 \sin \omega t$$

kmitny, uzly (zhuštění, zředění)



<http://saaphysics.com/PhysicsText/Ch1Sect2.htm>



$$n \frac{\lambda}{2} = L, n = 1, 2, \dots$$

$$(2n - 1) \frac{\lambda}{4} = L, n = 1, 2, \dots$$

$$f_n = n \frac{c}{2L} = n f_1, n = 1, 2, \dots$$

$$f_n = (2n - 1) \frac{c}{4L} = (2n - 1) f_1, n = 1, 2, \dots$$