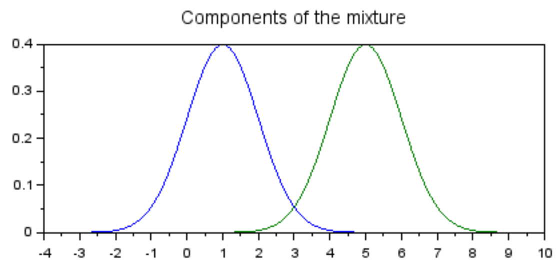
Simple mixture estimation in detail

Components: Scalar normal models with $r = 1$ (known)

$$f_1(x|\Theta_1) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(x-\Theta_1)^2\right\},\quad f_2(x|\Theta_1) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(x-\Theta_2)^2\right\}$$



Θ_1 and Θ_2 are unknown.

Preliminaries

For a single model we have

$$S_t = S_{t-1} + x_t$$
$$\kappa_t = \kappa_{t-1} + 1$$

$$\theta = \frac{S_t}{\kappa_t}$$

Initialization

Initial parameters for components

$$\theta_1 = 2,\quad \theta_2 = 7$$

Initial statistics for components (n_0 strength of prior, say $n_0 = 10$

$$\kappa_1 = n_0 = 10,\quad \kappa_2 = n_0 = 10$$
$$S_1 = \theta_1\kappa_1 = 10,\quad S_2 = \theta_2\kappa_2 = 70$$

which corresponds to prior parameters.

Data: $x = 2.7,\ 0.6,\ 4.9,\ \cdots$

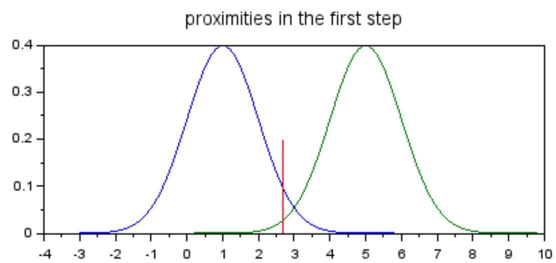
Time loop

- $t = 1$

– *proximity*

$$f_1(x_1|\theta_1) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[(x_1 = 2.7) - (\theta_1 = 1)\right]^2\right\} = 0.094$$

$$f_2(x_1|\theta_2) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[(x_1 = 2.7) - (\theta_2 = 5)\right]^2\right\} = 0.028$$



– *weights*

$$w = [0.769,\ 0.231]$$

$\cdots$ the data $x_1 = 2.7$ belongs more to the first component.

– *statistics update*

$$\kappa_1 = 10 + 0.769 = 10.769;\ \ \kappa_2 = 10 + 0.231 = 10.231$$

$$S_1 = 10 + 2.7 \cdot 0.769 = 12.076;\ S_2 = 70 + 2.7 \cdot 0.231 = 70.624$$

– *re-estimate of parameters*

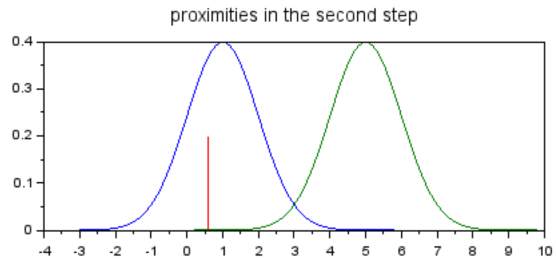
$$\theta_1 = 12.0763/10.769 = 1.121;\ \ \theta_2 = 70.624/10.231 = 6.903$$

- $t = 2$

– *proximity*

$$f_1(x_2|\theta_1) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[(x_2 = 0.6) - (\theta_1 = 1.121)\right]^2\right\} = 0.348$$

$$f_2(x_2|\theta_2) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[(x_2 = 0.6) - (\theta_2 = 6.903)\right]^2\right\} = 9.422 \cdot 10^{-10}$$


proximities in the second step
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10

– *weights*

$$w = [1,\ 0]$$

...