Estimation of binomial model

Model

$$f\left(y|p,N\right)=\binom{N}{y}p^{y}\left(1-p\right)^{N-y}$$

$y=0,1,2\cdots N,\ \ N\,i\mathrm{nteger,fixed.}$

Posterior

$$f\left(p|y\left(t\right)\right)=\prod_{i=1}^{t}\binom{N}{y_i}p^{S_t}\left(1-p\right)^{N\kappa_t-S_t}$$

Statistics update

$$S_t=S_{t-1}+y_t$$

$$\kappa_t=\kappa_{t-1}+1$$

Point estimate

$$\hat{p}_t=\frac{S_t}{N\kappa_t}=\bar{y}$$

Derivation

Likelihood

$$L_t\left(p\right)=\binom{N}{y_1}p^{y_1}\left(1-p\right)^{N-y_1}\binom{N}{y_2}p^{y_2}\left(1-p\right)^{N-y_2}\cdots=$$

$$=\binom{N}{y_1}\binom{N}{y_2}p^{y_1+y_2}\left(1-p\right)^{2N-\left(y_1+y_2\right)}\cdots\rightarrow$$

$$\to \text{const}.p^{S_t}\left(1-p\right)^{\kappa_t N - S_t}$$

where κ_t is the counter of data and $S_t = \sum_{i=1}^{t} y_i$ is the sum of samples.

Point estimate $\hat{p}_t = \arg\max_p L_t\left(o\right)$

$$\frac{d}{dp}L_t\left(p\right) = const.\frac{d}{dp}p^{S_t}\left(1-p\right)^{\kappa_t N - S_t} = 0$$

$$S_t p^{S_t - 1}\left(1-p\right)^{\kappa_t N - S_t} - p^{S_t}\left(\kappa_t N - S_t\right)\left(1-p\right)^{\kappa_t N - S_t}$$

$$S_t - S_t p = p\kappa_t N - S_t p$$

$$\hat{p}_t = \frac{S_t}{\kappa_t N}$$

Generation

$$b = \text{binomial(p,N-1)}$$

$$x = \text{sum(cumsum(b)} < \text{randu())}$$

Program

```
// T15modBin.sce
// ESTIMATION OF BINOMIAL/BERNOULLI MODEL
// Experiments
```

```
// - change parameters of the model
// ------------------------------------------------------------------
exec("ScIntro.sce",-1), mode(0)

p=.8; N=10;                             // params of binomial distribution
nd=1000;                                // dimensions of generated matrix

for t=1:nd
  x(t)=sampBin(p,N);                    // generation
end

S=80; ka=10;                            // prior information
for t=1:nd
  S=S+x(t);
  ka=ka+1;
  pE(t)=S/(N*ka);
end

// Results
plot(pE)
plot([nd/2 nd],[p p],':r')
xlabel('time','fontsize',4,'fontname','times')
ylabel('estimates','fontsize',4,'fontname','times')
```

```
title('Time evolution of the estimated parameter','fontsize',5,'fontname','times')
legend('estimats','true value');

// Remark
// For Bernoulli distribution set N = 1.
```