## Poisson model

Model

$$f\left(y|\lambda\right)=\exp\left\{-\lambda\right\}\frac{\lambda^{y}}{y!},\ \ y=0,1,2,\cdots,\ \lambda>0$$

Likelihood

$$L\left(\lambda\right)\propto\left(\exp\left\{-\lambda\right\}\right)^{\kappa_t}\frac{\lambda^{S_t}}{\prod y_i!}$$

with statistics $S_t$ and $\kappa_t$.

Update of statistics

$$S_t=S_{t-1}+y_t$$

$$\kappa_t=\kappa_{t-1}+1$$

Point estimate of $\lambda$

$$\hat{\lambda}_t=\frac{S_t}{\kappa_t}$$

Program and its description

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**Derivation**

Product of models for $y_1, y_2$

$$\exp\{-\lambda\}\frac{\lambda^{y_1}}{y_1!}\exp\{-\lambda\}\frac{\lambda^{y_2}}{y_2!} = (\exp\{-\lambda\})^2\frac{\lambda^{y_1+y_2}}{y_1!y_2!}$$

and generally (likelihood)

$$L(\lambda) = (\exp\{-\lambda\})^N\frac{\lambda^{y_1+y_2+\cdots+y_N}}{y_1!y_2!\cdots y_N!} = (\exp\{-\lambda\})^{\kappa_N}\frac{\lambda^{S_N}}{\prod y_i!}$$

Derivative of the likelihood je $\frac{\partial L}{\partial\lambda} = -\kappa_N\exp\{-\lambda\}^{\kappa_N-1}\frac{\lambda^{S_N}}{\prod y_i!} + (\exp\{-\lambda\})^{\kappa_N}\frac{S_N}{\prod y_i!}\lambda^{S_n-1} = 0$

$$\kappa_N\lambda = S_N$$

From which

$$\hat{\lambda} = \frac{S_N}{\kappa_N}$$