Mixture with state-space components

State estimation with single model

Notation $d_t = \{u_t, y_t\}$.

For a single state-space model we have

$$f(x_t|d(t)) \propto \int_{x^*_{t-1}} f(y_t, x_t, x_{t-1}|u_t, d(t-1))\, dx_{t-1} =$$

$$= \int_{x^*_{t-1}} f(y_t|x_t, u_t)\, f(x_t|x_{t-1}, u_t)\, f(x_{t-1}|d(t-1))\, dx_{t-1}$$

State estimation with mixture model

For one component it is the same, only all indexed by the pointer value $(c_t = j)$.

$$f_j(x_t|d(t)) \propto \int_{x^*_{t-1}} f_j(y_t|x_t, u_t)\, f_j(x_t|x_{t-1}, u_t)\, f_j(x_{t-1}|d(t-1))\, dx_{t-1}$$

It means, that each component has its own state which is separately estimated.

The proximity measures the discrepancy between the estimated model of a variable and the reality represented by t
measured value of this variable. The only variable that can be measured during the state estimation is the output

(the input is assumed to be generated by us). So, the model is $f_j\left(y_t|u_t,d\left(t-1\right)\right)$ which is provided by the Kalman filter procedure. This model also reflects the correctness of the state estimate.

From this point of view, the proximities are

$$q_j = f_j\left(y_t|u_t,d\left(t-1\right)\right)$$

with the inserted value of the measured output y.

Another way of defining the proximity is to use the point estimate of $y=\hat{y}$ (from the above pdf) and to base the proximity on a suitable function of the prediction error $ep_j=\hat{y}_j-yp_j$ like e.g. the following one

$$q_j = \frac{1}{\left(\hat{y}_j-yp_j\right)^2}.$$

The final state estimate is combined from component ones with the weights defined as normalized proximities.