Mixture estimation

As it has been already mentioned, mixtures are composed of a collection of single models (components) and a pointer whose values indicate the active component. The parameters of the components and the pointer model as well as the values of the pointer are estimated.

The pointer variable c_t can be assumed to have uniform distribution (which means, that all components have the same chance to be active at a given time instant). Then its model is

$$f\left(c_t = j | d\left(t-1\right)\right) = f\left(c_t = j\right) = \frac{1}{n_c}$$

where $d\left(t-1\right)$ denotes all old variables (usually y and u or v) and n_c is number of components.

The estimate of the pointer is given by the pdf

$$f\left(c_t = j | d\left(t\right)\right) = w_{j;t}$$

where the information of all measured data (including the present time instant) is used. The vector w_t is a weighting vector and its entry $w_{j;t}$ is a probability that the j-th component is active at the time instant t. It is proportional to so called proximity, which is th value of the component model with inserted measured data and the actual point estimates of model parameters

$$f\left(c_t = j | d\left(t\right)\right) \propto f_j\left(y_t | \psi_t, \hat{\Theta}_{t-1}\right)$$

The components are models or which (in a single form) we know estimation (it is th form of statistics, their update and construction of th point estimates of parameters).

Example

Let us have components to be estimated whose statistics are $S_{j;t}$ and $\kappa_{j;t}$ with the update (j stands for components)

$$S_{j;t}\kappa_{j;t} = S_{j;t-1} + y_t, \quad = \kappa_{j;t-1} + 1 \tag{1}$$

and the point estimate is

$$\hat{\Theta}_j = \frac{S_{j;t}}{\kappa_{j;t}}$$

which is the most frequent situation. What is important is that the statistics are updated by adding data: the first one adds d_t and the second one adds 1.

Now, the update of the components is the same, only the data are added with the weight w, i.e.

$$S_{j;t}\kappa_{j;t} = S_{j;t-1} + w_{j;t}y_t \quad = \kappa_{j;t-1} + w_{j;t} \tag{2}$$

The point estimates are computed in the same way.

Algorithm of mixture estimation

1. Initialize all components - i.e. set initial statistics for all components and construct point estimates of parameters
2. Measure new data record
3. Compute proximities
4. Normalize proximities to sum equal to one - weights

5. Update all statistics with the measured data and the constructed weights (2)
6. Compute point estimates of the component parameters
7. Go to 2.

The algorithm is demonstrated in the following program (for static normal mixture with known component variances).

Program and its description

Back to Main