Rayleigh Model

This model describes continuous nonnegative variable. Its pdf is

$$f(y|r) = \frac{y}{r}\exp\left(-\frac{y^2}{2r}\right),\ r > 0$$

Posterior

$$f(r|y(t)) = P_t r^{-\kappa_t}\exp\left(-\frac{S_t}{2r}\right)$$

where $P = \prod_{i=1}^{t} y_t$ and $S_t = \sum_{i=1}^{t} y_t^2$

Update (P is not needed for point estimation)

$$S_t = S_{t-1} + y_t^2$$
$$\kappa_t = \kappa_{t-1} + 1$$

Point estimate

$$\hat{r}_t = \frac{S_t}{2\kappa_t}$$

Generation[1]

$$y = \sqrt{-2r\ln(u)}$$

where u is $U(0,1)$.

Program and its description Back to Main

[1]From Wikipedia: https://en.wikipedia.org/wiki/Rayleigh_distribution

Derivation

Product

$$\frac{y_1}{r}\exp\left(-\frac{y_1^2}{2r}\right)\frac{y_2}{r}\exp\left(-\frac{y_2^2}{2r}\right) = y_1 y_2 r^{-2}\exp\left(-\frac{y_1^2+y_2^2}{2r}\right)$$

$\rightarrow$ posterior

Derivative

$$\frac{d}{dr}\frac{P_t}{r^{\kappa_t}}r^{-\kappa_t}\exp\left(-\frac{S_t}{2r}\right) =$$

$$= -P_t\kappa_t r^{-\kappa_t-1}\exp\left(-\frac{S_t}{2r}\right) + P_t r^{-\kappa_t}\exp\left(-\frac{S_t}{2r}\right)\frac{S_t}{2r^2} = 0$$

$$\kappa_t = r\frac{S_t}{2r^2} \;\rightarrow\; \hat{r}_t = \frac{S_t}{2\kappa_t}$$