Scalar regression model

Equation of scalar regression model

Pdf for scalar regression model is generated by the equation

$$y_t = b_0 u_t + c_0 v_t + a_1 y_{t-1} + b_1 u_{t-1} + c_1 v_{t-1} + \cdots + a_1 y_{t-n} + b_1 u_{t-n} + c_1 v_{t-n} + k + e_t \tag{1}$$

where u, v, y are variables: control, input, target, b, c, a are corresponding regression coefficients, k is constant and e is normal white noise with zero expectation and variance r.

The denotation

$\psi_t = [u_t, v_t, y_{t-1}, u_{t-1}, v_{t-1}, \cdots, y_{t-n}, u_{t-n}, v_{t-n}, 1]$ - regression vector

$\theta = [b_0, c_0, a_1, b_1, c_1, \cdots, a_n, b_n, c_n, k]$ - vector of parameters

enables to write the model equation in the vector form

$$y_t = \psi_t^{'} \theta + e_t$$

Estimation of scalar regression model

Extended regression vector

$$\Psi_t = \left[y_t, \psi_t^{'}\right]^{'}$$

Statistics and update

- information matrix V with rank $\dim(\Psi) \times \dim(\Psi)$

$$V_t = V_{t-1} + \Psi_t \Psi_t^{'}$$

- counter κ

$$\kappa_t + \kappa_{t-1} + 1$$

Point estimates

$$\hat{\theta}_t = V_p^{-1} V_{yp}$$

where

$$V = \begin{bmatrix} V_y & V_{yp}^{'} \\ V_{yp} & V_p \end{bmatrix}$$

and V_y is a scalar.

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Estimation of static regression model with known variance

The model is

$$y_t = k + e_t$$

with model pdf

$$\frac{1}{\sqrt{2\pi r}} \exp\left(-\frac{1}{2r}\left(y_t - k\right)^2\right)$$

and r is known constant.

Likelihood

$$L_t\left(k\right) \propto \exp\left(-\frac{1}{2r}\sum_{i=1}^{t}\left[y_t^2 - 2ky_t\right] + tk^2\right) = \exp\left(-\frac{1}{2r}\underbrace{\sum_{i=1}^{t} y_t^2}_{R_t} - 2k\underbrace{\sum_{i=1}^{t} y_t}_{S_t} + \underbrace{t}_{\kappa_t} k^2\right)$$

Update

$$R_t = R_{t-1} + y_t^2$$

$$S_t = S_{t-1} + y_t$$

$$\kappa_t = \kappa_{t-1} + 1$$

Derivative of the likelihood

$$\frac{d}{dk}\exp\left(-\frac{1}{2r}R_t - 2kS_t + \kappa_t k^2\right) = \exp\left(-\frac{1}{2r}R_t - 2kS_t + \kappa_t k^2\right)\left(-2S_t + 2k\kappa_t\right) = 0$$

$\rightarrow S_t = k\kappa_t$ and

$$\hat{k}_t = \frac{S_t}{\kappa_t}$$

Remark

The statistics R is not needed in this case.

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Least squares estimation

Example for the first order model and N data records

$$y_t = b_0 u_t + a_t y_{t-1} + b_1 u_{t-1} + k + e_t$$

Construct

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_N \end{bmatrix}, \quad X = \begin{bmatrix} u_1 & y_0 & u_0 & 1 \\ u_2 & y_1 & u_1 & 1 \\ \dots & \dots & \dots & \dots \\ u_n & y_{N-1} & u_{N-1} & 1 \end{bmatrix}$$

Point estimate

$$\hat{\theta}_t = \left(X'X\right)^{-1} X'Y$$

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