Introduction

On-line (recursive) estimation consists in sequential update of the statistics and if needed ongoing computation of th point estimates of the parameters. Each step, the newly measured data are added and used for improving the estimates. The process starts with a prior statistics to be recomputed at the first step. If there is no prior information the prior statistics can be constructed so they give uniform prior distribution of parameters (i.e. each value of them i equally probable). However, it is highly unreasonable, from the following reasons:

1. Always, there is some prior information - stronger or weaker, but it is.
2. With uniform prior density of parameters the initial estimates can be very wild and for example if the model is used for control, they can be the cause of the failure of the whole control process.
3. At the beginning of estimation we use only few data (at the first step only one data record). This record is under uncertainty and thus it can determine parameters that are far from the true values (imagine e.g. tossing a coin). Then the estimates must be dragged back (with partially fixed statistics and they do not need to reach the true positions).

To prevent these inconveniences, we can

- set the initial statistics so that they produce reasonable point estimates of the parameters,
- tighten the initial estimates appropriately so that they hold on for a while and can be moved only by a larger portion of data.

These two steps are universal (for arbitrary model), but for each one the way how to do it is a it different. We shall demonstrate the way on the categorical model describing tossing of a coin.

Example

Let us have a discrete variable $y_t \in \{0,1\}$, $t = 1,2,\cdots,N$ describing tossing a coin. The value $y_t = 1$ represents the result "head" and $y_t = 0$ is "tail". We have tossed the coin 5 times (N=5) and obtained the data

$$1\ 1\ 0\ 1\ 0$$

Perform step by step the estimation of the probability $P\,(\text{"}head\text{"})$. We know that in reality the the probability is $P = 0.5$.

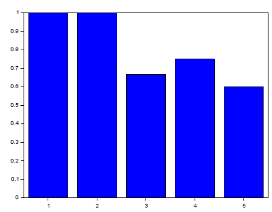
The estimation can be performed according to the statistical definition of probability

$$P = \frac{\text{number of heads}}{\text{number of experiments}}$$

which is listed in the following table ($\hat{p}$ is the estimate of the probability)

t	numb. of heads	numb. of experiments	$\hat{p}$
1	1	1	1
2	2	2	1
3	2	3	$\frac{2}{3}$
4	3	4	$\frac{3}{4}$
5	3	5	$\frac{3}{5}$

We can see that at the beginning the estimates are totally wrong and only gradually they are refined. The estimates evolution is



Now, with our knowledge (that the coin is correct), we can construct so called fictitious prior data expressing out knowledge, e.g.

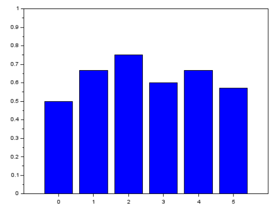
$$1 \; 0$$

and use them before the estimation starts. We obtain

t	numb. of heads	numb. of experiments	$\hat{p}$
0	1	2	$\frac{1}{2}$
1	2	3	$\frac{2}{3}$
2	3	4	$\frac{3}{4}$
3	3	5	$\frac{3}{5}$
4	4	6	$\frac{2}{3}$
5	4	7	$\frac{4}{7}$

where time $t = 0$ represents prior data and their use is counted as the first two experiments (we inserted two measurements 1 and 0).

The evolution is

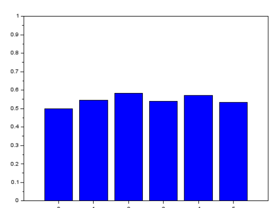


and it is much better then the previous case.

Finally, we can add not only two fictitious data but more, if we are convinced in our prior belief. Say that we decided to add five results 1 and five results 0 (for the probability $P = 0.5$). In our table, in the row 0, it will add 5 in the column “heads” and 10 in “results. So, we have

t	numb. of heads	numb. of experiments	$\hat{p}$
0	5	10	$\frac{1}{2}$
1	6	11	$\frac{6}{11}$
2	7	12	$\frac{7}{12}$
3	7	13	$\frac{7}{13}$
4	8	14	$\frac{4}{7}$
5	8	15	$\frac{8}{15}$

and the evolution is



and it respect relatively strongly our prior belief.

This example illustrates the commonly valid rules for inserting the prior belief into the on-line parameter estimation:

1. The ratio of the values of fictitious prior data express the values of the parameters.
2. The magnitude of the values determines the strength of our belief.

Program

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Introduction
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On-line (recursive) estimation consists in sequential update of the statistics
 and if needed ongoing computation of th point estimates of the parameters.
 Each step, the newly measured data are added and used for improving the
 estimates.
 The process starts with a prior statistics to be recomputed at the first
 step.
 If there is no prior information the prior statistics can be constructed
 so they give uniform prior distribution of parameters (i.e.
 each value of them i equally probable).
 However, it is highly unreasonable, from the following reasons:
\end_layout

\begin_layout Enumerate
Always, there is some prior information - stronger or weaker, but it is.
\end_layout

\begin_layout Enumerate
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 This record is under uncertainty and thus it can determine parameters that
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 Then the estimates must be dragged back (with partially fixed statistics
 and they do not need to reach the true positions).

\end_layout

\begin_layout Standard
To prevent these inconveniences, we can
\end_layout

\begin_layout Itemize
set the initial statistics so that they produce reasonable point estimates
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\begin_layout Itemize
tighten the initial estimates appropriately so that they hold on for a while
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These two steps are universal (for arbitrary model), but for each one the
 way how to do it is a it different.
 We shall demonstrate the way on the categorical model describing tossing
 of a coin.
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\begin_layout Subsubsection
Example
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\begin_layout Standard
Let us have a discrete variable
\begin_inset Formula $y_{t}\in\left\{ 0,1\right\} ,$
\end_inset


\begin_inset Formula $t=1,2,\cdots,N$
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 describing tossing a coin.
 The value
\begin_inset Formula $y_{t}=1$
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 represents the result
\begin_inset Quotes eld
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head
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 and
\begin_inset Formula $y_{t}=0$
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 is
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tail
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.
 We have tossed the coin 5 times (N=5) and obtained the data
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\begin_layout Standard
\align center
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1 1 0 1 0
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\begin_layout Standard
Perform step by step the estimation of the probability
\begin_inset Formula $P\left("head"\right)$
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.
 We know that in reality the the probability is
\begin_inset Formula $P=0.5$
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\begin_layout Standard
The estimation can be performed according to the statistical definition
 of probability
\begin_inset Formula
\[
P=\frac{\mbox{number of heads}}{\mbox{number of experiments}}
\]

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which is listed in the following table (
\begin_inset Formula $\hat{p}$
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 is the estimate of the probability)
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We can see that at the beginning the estimates are totally wrong and only
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 The estimates evolution is
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Now, with our knowledge (that the coin is correct), we can construct so
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and use them before the estimation starts.
 We obtain
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where time
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 represents prior data and their use is counted as the first two experiments
 (we inserted two measurements 1 and 0).

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The evolution is
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\align left
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\begin_layout Standard
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ty
\begin_inset Formula $P=0.5$
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heads
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 and 10 in
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results.
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\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{1}{2}$
\end_inset


\end_layout

\end_inset
</cell>
</row>
<row>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
1
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
\begin_inset Text
```

```
\begin_layout Plain Layout
6
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
11
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" leftline="true" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{6}{11}$
\end_inset


\end_layout

\end_inset
</cell>
</row>
<row>
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\begin_inset Text

\begin_layout Plain Layout
2
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
```

```
\begin_inset Text

\begin_layout Plain Layout
7
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
12
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\end_inset
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\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{7}{12}$
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\end_layout

\end_inset
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\begin_inset Text

\begin_layout Plain Layout
3
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\end_inset
</cell>
```

```
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\begin_layout Plain Layout
7
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\end_inset
</cell>
<cell alignment="center" valignment="top" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
13
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</cell>
<cell alignment="center" valignment="top" leftline="true" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{7}{13}$
\end_inset


\end_layout

\end_inset
</cell>
</row>
<row>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
4
\end_layout

\end_inset
```

```
</cell>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
8
\end_layout

\end_inset
</cell>
<cell alignment="center" valignment="top" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
14
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<cell alignment="center" valignment="top" leftline="true" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{4}{7}$
\end_inset


\end_layout

\end_inset
</cell>
</row>
<row>
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\begin_inset Text

\begin_layout Plain Layout
5
\end_layout
```

```
\end_inset
</cell>
<cell alignment="center" valignment="top" rightline="true" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
8
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\end_inset
</cell>
<cell alignment="center" valignment="top" usebox="none">
\begin_inset Text

\begin_layout Plain Layout
15
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\begin_inset Text

\begin_layout Plain Layout
\begin_inset Formula $\frac{8}{15}$
\end_inset


\end_layout

\end_inset
</cell>
</row>
</lyxtabular>

\end_inset


\end_layout
```

```
\begin_layout Standard
and the evolution is
\end_layout

\begin_layout Standard
\align center
\begin_inset Graphics
filename est3.png
lyxscale 50
scale 20

\end_inset


\end_layout

\begin_layout Standard
and it respect relatively strongly our prior belief.
\end_layout

\begin_layout Standard
This example illustrates the commonly valid rules for inserting the prior
 belief into the on-line parameter estimation:
\end_layout

\begin_layout Enumerate
The ratio of the values of fictitious prior data express the values of the
 parameters.
\end_layout

\begin_layout Enumerate
The magnitude of the values determines the strength of our belief.

\end_layout

\begin_layout Standard
Program
\end_layout

\begin_layout Standard
```

```
\begin_inset CommandInset include
LatexCommand verbatiminput
filename "d2_init0.lyx"

\end_inset


\end_layout

\begin_layout Subsubsection
\begin_inset Note Note
status collapsed

\begin_layout Subsubsection
Inserting prior belief for estimation of expectation
\end_layout

\begin_layout Plain Layout
For estimation of the expectation (i.e.
 for models exponential, Bernoulli, binomial and similar ones), the statistics
 (sum and counter) and their update are
\begin_inset Formula
\[
S_{t}=S_{t-1}+y_{t}
\]

\end_inset


\begin_inset Formula
\[
\kappa_{t}=\kappa_{t-1}+1
\]

\end_inset


\end_layout

\begin_layout Plain Layout
```

```
Here we can proceed as follows: let
\begin_inset Formula $p_{0}$
\end_inset

 is our guess of the parameter.
 Then set
\begin_inset Formula
\[
\kappa_{0}=N_{0}
\]

\end_inset



\begin_inset Formula
\[
S_{0}=p_{0}\kappa_{0}
\]

\end_inset

where
\begin_inset Formula $N_{0}$
\end_inset

 is the number of fictitious data expressing the strength of our belief
 in the value
\begin_inset Formula $p_{0}$
\end_inset


.
\end_layout

\begin_layout Plain Layout
Then
\begin_inset Formula
\[
\hat{p}=\frac{S_{0}}{\kappa_{0}}=\frac{p_{0}\kappa_{0}}{\kappa_{0}}=p_{0}
\]
```

```
\end_inset

 independently of
\begin_inset Formula $N_{0}$
\end_inset

 which is free for setting the strength.
\end_layout

\begin_layout Subsubsection
Inserting prior belief for regression models
\end_layout

\begin_layout Plain Layout
For scalar regression model it holds
\begin_inset Formula
\[
V_{t}=V_{t-1}+\Psi_{t}\Psi_{t}^{'}
\]

\end_inset

 where
\begin_inset Formula $\Psi_{t}=\left[y_{t},\psi_{t}^{'}\right]^{'}$
\end_inset

 is extended regression vector with
\begin_inset Formula $\psi_{t}$
\end_inset

 being the regression vector.
 The point estimate is
\begin_inset Formula
\[
\hat{\theta}=V_{\psi}^{-1}V_{y\psi}
\]

\end_inset

with the partitioning
```

```
\begin_inset Formula
\[
V=\left[\begin{array}{cc}
V_{y} & V_{y\psi}^{'}\\
V_{y\psi} & V_{\psi}
\end{array}\right]
\]

\end_inset

 with the dimensions given by the extended regression vector.
\end_layout

\begin_layout Plain Layout
So, if we want the parameters to be
\begin_inset Formula $\theta_{0}$
\end_inset

 then we set
\end_layout

\begin_layout Plain Layout
\begin_inset Formula
\[
\tilde{V}=\left[\begin{array}{cc}
1 & \theta_{0}^{'}\\
\theta_{0} & I
\end{array}\right]
\]

\end_inset

with
\begin_inset Formula $I$
\end_inset

 being a unit matrix, and the initial statistics are
\begin_inset Formula
\[
\kappa_{0}=N_{0}
```

```
\]

\end_inset


\begin_inset Formula
\[
V_{0}=\kappa_{0}\tilde{V}
\]

\end_inset


\end_layout

\begin_layout Plain Layout
Then we obtain
\begin_inset Formula
\[
\hat{\theta}=V_{\psi}^{-1}V_{y\psi}=\left(\kappa_{0}I\right)^{-1}\times\kappa_{0}\theta_{0}=\theta_{0
\]

\end_inset

and again
\begin_inset Formula $N_{0}$
\end_inset

 express our belief in the estimate
\begin_inset Formula $\theta_{0}.$
\end_inset


\end_layout

\begin_layout Plain Layout
Program
\end_layout

\begin_layout Plain Layout
```

```
\begin_inset CommandInset include
LatexCommand verbatiminput
filename "Test4b.sce"

\end_inset


\end_layout

\end_inset


\end_layout

\end_body
\end_document
```