State estimation - unknown model parameters

Known parameters of the state-space model required by the Kalman filter procedure are very strong assumption that is is rarely fulfilled. If some model parameters are unknown, we can define a new state composed of the old state and the unknown parameters. By this step, the model becomes nonlinear and we mus linearized it as mentioned before.

Example

Let us have the model

$$
\begin{aligned}
x_t &= a x_{t-1} + u_{t-1} + w_t \\
y_t &= b x_t + v_t
\end{aligned}
$$

where a and b are unknown model parameters.

We define new state

$$
X_t = \begin{bmatrix} X_{1;t} \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} x_t \\ a \\ b \end{bmatrix}
$$

with the model

$$
\begin{bmatrix} X_{1;t} \\ X_{2;t} \\ X_{3;t} \end{bmatrix} = \underbrace{\begin{bmatrix} X_{2;t-1} X_{1;t-1} + u_{t-1} \\ X_{2;t-1} \\ X_{3;t-1} \end{bmatrix}}_{g}
$$

$$
y_t = \underbrace{X_{3;t} X_{1;t}}_{h}
$$

Linearization gives

$$X_t = \bar{M} X_{t-1} + F + W_t$$
$$y_t = \bar{A} X_t + G + V_t$$

where

$$\bar{M} = \begin{bmatrix} X_{2;t-1} & X_{1;t-1} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad F = g - \bar{M} X_{t-1}$$
$$\bar{A} = [X_{3;t},\ 0,\ X_{1;t}], \quad G = h - \bar{A} X_t$$

This model can be directly used by Kalman filter.

Program and its description

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