Multivariate regression model and its estimation

Equation

An example of the model is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u_t + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_t + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_t$$

where the regression vector $\psi_t = [y_{t-1}, u_t, v_t, 1]$ a parameters $\theta = \{a, b, c, k\}$, + r covariance matrix of e_t.

Shortly

$$y_t = ay_{t-1} + bu_t + cv_t + k + e_t$$

and in the matrix form

$$y_t = \theta\psi_t + e_t$$

$\psi = [y_{1;t-1}, y_{2;t-1}, u_t, v_{1;t}, v_{2;t}, v_{3;t}, 1]'$

$\theta = \begin{bmatrix} a_{11} & a_{12} & b_1 & c_{11} & c_{12} & c_{13} & k_1 \\ a_{21} & a_{22} & b_2 & c_{21} & c_{22} & c_{23} & k_2 \end{bmatrix}$

Estimation (Bayes)

Extended regression vector (for the example)

$$\Psi_t = [y_{t;t},\, y_{2;t},\, y_{1;t-1},\, y_{2;t-1},\, u_t,\, v_{1;t},\, v_{2;t},\, v_{3;t},\, 1]'$$

Information matrix

$$V = \begin{bmatrix} V_y & V_{yp}' \\ V_{yp} & V_p \end{bmatrix}$$

where V_y is matrix 2×2, V_{yp} is matrix 2×7 and V_p is matrix 7×7.

Point estimates

$$\hat{\theta}_t = V_p^{-1} V_{yp}$$

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Estimation (Least squares)

For the example: create

$$Y = \begin{bmatrix} y_{1;1} & y_{2;1} \\ y_{1;2} & y_{2;2} \\ y_{1;3} & y_{2;3} \\ \cdots & \cdots \\ y_{1;N} & y_{2;N} \end{bmatrix}, \quad X = \begin{bmatrix} y_{1;0} & y_{2;0} & u_1 & v_{1;1} & v_{2;1} & v_{3;1} & 1 \\ y_{1;1} & y_{2;1} & u_2 & v_{1;2} & v_{2;2} & v_{3;2} & 1 \\ y_{1;2} & y_{2;2} & u_3 & v_{1;3} & v_{2;3} & v_{3;3} & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{1;N-1} & y_{2;N-1} & u_N & v_{1;N} & v_{2;N} & v_{3;N} & 1 \end{bmatrix}$$

Point estimates are

$$\hat{\theta} = (X'X)^{-1} X'Y$$

Remark

The estimate entries correspond the the parameter θ shown with the equation above.

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