State estimation - nonlinear model

Nonlinear model is given by nonlinear functions g and h, as follows

$$x_t = g\left(x_{t-1}, u_t\right) + w_t$$
$$y_t = h\left(x_t, u_t\right) + v_t$$

To be able to use Kalman filter, we need to linearize it. We use the first two items of Taylor expansion of the functions g and h.

For a general value x the expansion around the point $\hat{x}_{t-1}$ (last point estimate) reads

$$g\left(x, u_t\right) \doteq g\left(\hat{x}_{t-1}, u_t\right) + g'\left(\hat{x}_{t-1}, u_t\right)\left(x - \hat{x}_{t-1}\right)$$

$$h\left(x, u_t\right) \doteq h\left(\hat{x}_t, u_t\right) + h'\left(\hat{x}_t, u_t\right)\left(x - \hat{x}_t\right)$$

where g' and h' denote derivative of g and h.

We obtain linear model

$$\begin{aligned} x_t &= \bar{M} x_{t-1} + F + w_t \\ y_t &= \bar{A} x_t + G + v_t \end{aligned}$$

where

$$\bar{M} = g'\left(\hat{x}_{t-1}, u_t\right), \qquad F = g\left(\hat{x}_{t-1}, u_t\right) - g'\left(\hat{x}_{t-1}, u_t\right)\hat{x}_{t-1},$$

$$\bar{A} = h'\left(\hat{x}_t, u_t\right), \qquad G = h\left(\hat{x}_t, u_t\right) - h'\left(\hat{x}_t, u_t\right)\hat{x}_t.$$

Example

The state-space model is

$$\begin{aligned} x_{1;t} &= x_{1;t-1}x_{2;t-1} + u_{t-1} + w_{1;t} \\ x_{2;t} &= 0.5x_{1;t-1} + 0.8x_{2;t-1} - u_{t-1} + w_{2;t} \\ y_t &= x_{1;t} \end{aligned}$$

Then

$$\bar{M} = \left[\begin{array}{cc} x_2 & x_1 \\ 0.5 & 0.8 \end{array}\right]_{t-1}, \quad F = \left[\begin{array}{c} x_1x_2 + u \\ 0.5x_1 + 0.8x_2 - u \end{array}\right]_{t-1} + M\bar{x}_{t-1}$$

$$\bar{A} = [1,\, 0]\,, \quad G = 0$$

Program and its description

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