# Semestral work for MMJ 2020/2021

Try all but respond only on boldface.

## Example 1

1. Part

   (a) Simulate data from the 2nd order regression model with coefficients

   $$b_0 = 1,\ a_1 = 0.3,\ b_1 = 0.4,\ a_2 = 0.5,\ b_2 = 0.1,\ k = -0.1$$

   and standard deviation of the noise equal to $s = 0.01$. The input is a sinusoidal signal with added normal noise $w_t$

   $$u_t = \rho \sin(\omega t) + \vartheta w_t$$

   where $\rho = 0.1$ and and $\omega$ is set so that the input signal performs two periods on the simulation interval which is 200 steps. Plot the simulated data.

   (b) Perform estimation of the 1st order regression model of the form

   $$y_t = b_0 u_t + a_1 y_{t-1} + b_1 u_{t-1} + k + e_t$$

   using the simulated data. **Compare the simulated and estimated parameters**.

   (c) Using the simulated data and estimated model, perform zero-step prediction (i.e. at time $t$ make prediction $yp_t$ without using $y_t$).

   (d) Plot graph of $u$, $y$ and $yp$ and compute RPE (relative prediction error: variance$(y-yp)$/variance$(y)$ ).

   (e) **Make conclusions about the estimated parameters (in comparison with the simulated ones) and about the accuracy of prediction**.

2. Part - perform 1st Part but for estimation use 2nd order model.

3. Part - perform 1st and 2nd Part with the following combinations of parameters $s$ (standard deviation of the simulation model noise) and $\vartheta$ (amplitude of the noise added to control $u_t$)

| case | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $s$ | 0 | 0 | 0.01 | 10 |
| $\vartheta$ | 0 | 0.1 | 0.1 | 0.1 |

   For evaluation of the experiments use RPE. **Formulate theoretical conclusions and compare them with the results provided by experiments.**

## Example 2

Simulate 200 data items from discrete model $f(y_t|y_{t-1})$, $y \in \{1,2\}$ for which it holds that the probability that $y$ changes its value is 0.1.

Use the simulated data for ongoing estimation (continuously calculate and store point estimates of parameters) of the model with the same structure. Store only the first column of the estimates.

Show the evolution of the point estimates for the following cases of the statistics initiation

1. $V$=1e-8*eye(2,2) - practically no prior information,
2. $V$ =5*eye(2,2) - strong almost true information,
3. $V$ =0.5*ones(2,2) - weak false information,
4. $V$=5*ones(2,2) - strong false information.

**Compare the graphical results and comment them. Print also the final estimates and compare them**.

## Example 3

For 3rd order regression model

$$y_t = 0.8y_{t-1} + 0.2y_{t-2} + 0.1y_{t-3} + 0.1u_t + 0.4u_{t-1} + 0.1u_{t-2} - 0.2u_{t-3} + k + e_t$$

with $e_t \sim N\left(0, \sigma^2\right)$ with $\sigma = 0.1$ perform optimal control minimizing the criterion

$$E\left[\sum_{t=1}^{N} \left(y_t - s_r\right)^2 + \omega u_t^2 + \lambda \left(u_t - u_{t-1}\right)^2\right]$$

on control interval 100 steps.

Here $s$ is a setpoint in the form

$$s_t = \text{sign}\left(10 \sin\left(18\left(1:N\right)/N\right)\right)$$

with sign$(\cdot)$ is the sign function: sign$(x)$ is 1 for $x > 0$, is 0 for $x = 0$ and is -1 for $x < 0$.

**Compare the results for various combinations of values of penalizations $\omega$ and $\lambda$. Explain the results you obtain**.

Hint

As a basis for your work use the program T53ctrlX.sce.